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15/ENG03/022

1) $\frac{\partial y}{\partial t} + 3y = e^{-2t}$ Given that $t = 0, y = 2$

$y' + 3y = e^{-2t}$

$L[y'(t)] = sY(s) - y(0)$

$L(y(t)) = Y(s)$

$L[e^{-2t}] = \frac{1}{s+2}$

$= sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$Y(s)(s+3) - y(0) = \frac{1}{s+2}$

$Y(s)(s+3) - 2 = \frac{1}{s+2}$

$Y(s)(s+3) = \frac{1}{s+2} + \frac{2}{1}$

$Y(s)(s+3) = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{s+2} \times \frac{1}{s+3} = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

at $s = -3$

$2(-3) + 5 = A(-3+3) + B(-3+2)$

$-6 + 5 = -B$

$-1 = -B$

$B = 1$

at $s = -2$

$2(-2) + 5 = A(-2+3) + B(-2+2)$

$-4 + 5 = A$

$A = 1$

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$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$$

$$= e^{-2t} + e^{-3t}$$

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2) $3\frac{dy}{dt} - 6y = \sin 2t$.

$3y' - 6y = \sin 2t$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2}$

$= 3(sY(s) - y(0)) - 6Y(s) = \frac{2}{s^2+2^2}$

$= (3s-6)Y(s) - 3 = \frac{2}{s^2+4}$

$(3s-6)Y(s) = \frac{2}{s^2+4} + 3 = \frac{2}{(s+2)^2} + \frac{3}{1}$

$\frac{2+3(s+2)^2}{(s+2)^2}$

$Y(s) = \frac{2+3(s+2)^2}{(s+2)^2} \times \frac{1}{(3s-6)}$

$= \frac{2+3(s+2)^2}{(s+2)^3(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{3s-6}$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

at $s = -2$

$2+3(-2+2)^2 = A(-2+2)(3(-2)-6) + B(3(-2)-6) + C(-2+2)^2$

$2+12+14 = A(0)(-12) + B(-12) + C(0)$

$-12B = 28$

$B = -\frac{1}{6}$

$3s^2 + 12s + 14 = A(3s^2 - 12) + B(3s-6) + C(s^2+4s+4)$

$3A + C = 3$

$3B + 4C = 12$

$-12A - 6B + 4C = 14$

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$$3A + C = 3$$

$$A = \frac{3 - C}{3}$$

$$-12A - 6B + 4C = 14$$

$$-12\left(\frac{3 - C}{3}\right) - 6B + 4C = 14$$

$$-12 + 4C - 6B + 4C = 14$$

$$-6B + 8C = 26$$

$$-6\left(\frac{-1}{6}\right) + 8C = 26$$

$$8C = 26 - 1$$

$$C = \frac{25}{8}$$

$$A = \frac{3 - {}^{25}/_{8}}{3} = \frac{\frac{-1}{8}}{3} = \frac{-1}{24}$$

$$= \frac{-1/24}{s+2} + \frac{-1/6}{(s+2)^2} + \frac{25/8}{3s-6}$$

$$\mathcal{L}^{-1} = \frac{-1}{24}\left[\frac{1}{s+2}\right] - \frac{1}{6}\left[\frac{1}{(s+2)^2}\right] + \frac{25}{8}\left[\frac{1}{3(s-2)}\right]$$

$$= \mathcal{L}^{-1}\left[\frac{-1}{24}\left[\frac{1}{s+2}\right] - \frac{1}{6}\left[\frac{1}{(s+2)^2}\right] + \frac{25}{24}\left[\frac{1}{s-2}\right]\right]$$

$$= \frac{-1}{24} e^{-2t} - \frac{1}{6} t e^{-2t} + \frac{25}{24} e^{2t}$$

3) $\frac{dy}{dt} - 4y = 8$ at $t=0, y=2$

$L[y'] = sY(s) - y(0)$

$L(y) = Y(s)$

$L[8] = \frac{8}{s}$

$= sY(s) - y_0 - 4Y(s) = \frac{8}{s}$

$(s-4)Y(s) - 2 = \frac{8}{s}$

$(s-4)Y(s) = \frac{8}{s} + 2$

$Y(s) = \frac{8+2s}{s} \times \frac{1}{s-4}$

$= \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8+2s = A(s-4) + Bs$

at $s=4$

$8+2(4) = A(4-4) + 4B$

$16 = 4B$

$B = 4$

at $s=0$

$8+2(0) = A(0-4) + B(0)$

$8 = -4A$

$A = -2$

$= \frac{-2}{s} + \frac{4}{(s-4)}$

$= L^{-1}\left[\frac{-2}{s}\right] + L^{-1}\left[\frac{4}{s-4}\right]$

$= -2 + 4e^{4t}$

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4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

$y'' - 2y' + 5y = e^{2t}$

$\mathcal{L}(y''(t)) = s^2Y(t) - sy(0) - y'(0)$ $\quad \mathcal{L}[e^{2t}] = \frac{1}{s-2}$

$\mathcal{L}[y'(t)] = sY(t) - y(0)$

$\mathcal{L}[y(t)] = Y(t)$

$= s^2Y(t) - sy(0) - y'(0) - 2(sY(t) - y(0)) + 5Y(t) = \frac{1}{s-2}$

$= s^2Y(t) - sy(0) - y'(0) - 2sY(t) + 2y(0) + 5Y(t) = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(t) - sy(0) - y'(0) + 2y(0) = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(t) - 2s - 1 + 4 = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(t) - 2s + 3 = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(t) = \frac{1}{s-2} + 2s - 3$

$(s^2 - 2s + 5)Y(t) = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$(s^2 - 2s + 5)y(t) = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2s^2 - 7s + 7}{s-2}$

$Y(t) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A(s^2 - 2s + 5) + Bs + C(s-2)}{(s-2)(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$2s^2 - 7s + 7 \quad 2As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$A + B = 2 \quad \text{--- (1)}$

$-2A - 2B + C = -7 \quad \text{--- (2)}$

$5A - 2C = 7 \quad \text{--- (3)}$

$A + 3 = 2$

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$2(2)^2 - 7(2) + 7 = A(2^2 - 2(2) + 5)$

$8 - 14 + 7 = 5A$

$1 = 5A$

$A = 1/5$

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$A + B = 2$

$B = 2 - 1/5$

$\frac{10 - 1}{5} = \frac{9}{5}$

$5A - 2C = 7$

$5(1/5) - 2C = 7$

$1 - 2C = 7$

$-2C = 6$

$C = -3$

$= \frac{1/5}{s-2} + \frac{9/5 s - 3}{s^2 - 2s + 5}$

$= \frac{\frac{1}{5}}{(s-2)} + \frac{\frac{9}{5}s}{s^2 - 2s + 5} - \frac{3}{s^2 - 2s + 5}$

$= \frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left(\frac{s-1+1}{(s-1)^2+4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$

$= L^{-1}\left[\frac{1/5}{s-2} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} - \frac{1 \times 2/2}{(s-1)^2+2^2}\right) - \frac{3}{2}\left[\frac{2}{(s-1)^2+2^2}\right]\right.$

$= L^{-1}\left[\frac{1/5}{s-2} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left[\frac{2}{(s-1)^2+2^2}\right] - \frac{3}{2}\left[\frac{2}{(s-1)^2+2^2}\right.\right.\right.$

$= \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t \cos 2t + \frac{1}{2}e^t \sin 2t\right] - \frac{3}{2}\left[e^t \sin 2t\right]$

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5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0, y=0, y'=2$

$y'' - 6y' + 8y = e^{3t}$

$\mathcal{L}[y''(t)] = s^2Y(s) - sy(0) - y'(0)$

$\mathcal{L}[y'(t)] = sY(s) - y(0)$

$\mathcal{L}[y(t)] = Y(s)$

$\mathcal{L}[e^{3t}] = \frac{1}{s-3}$

$s^2Y(s) - sy(0) - y'(0) - 6(sY(s) - y(0)) + 8Y(s) = \frac{1}{s-3}$

$(s^2 - 6s + 8)Y(s) + (-s+6)y(0) - y'(0) = \frac{1}{s-3}$

$(s^2 - 6s + 8)Y(s) + (-s+6)0 - 2 = \frac{1}{s-3}$

$(s^2 - 6s + 8)Y(s) = \frac{1}{s-3} + 2$

$= \frac{1+2(s-3)}{s-3} = \frac{1+2s-6}{s-3} = \frac{-5+2s}{s-3}$

$Y(s) = \frac{-5+2s}{s-3} \times \frac{1}{s^2-6s+8}$

$= \frac{-5+2s}{(s-3)(s-4)(s-2)} = \frac{A}{(s-3)} + \frac{B}{s-4} + \frac{C}{s-2}$

$2s - 5 = A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)$

when $s = 4$

$2(4) - 5 = A(4-4)(4-2) + B(4-3)(4-2) + C(4-3)(4-4)$

$3 = 2B$

$B = 3/2$

when $s = 3$

$2(3) - 5 = A(3-4)(3-2) + B(3-3)(3-2) + C(3-3)(3-4)$

$6 - 5 = 2C$

$-1 = 2C$

$C = -1/2$

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When s = 3

$2(3) - 5 = A(3-4)(3-2) + B(3-3)(3-2) + C(3-3)(3-4)$

$1 = -A$

$A = -1$

$= \frac{-1}{s-3} + \frac{3/2}{s-4} + \frac{-1/2}{s-2}$

$= L^{-1}\left[\frac{-1}{s-3}\right] + \left[\frac{3/2}{s-4}\right] - \frac{\frac{1}{2}}{s-2}$

$= -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$

$= \frac{3}{2}e^{4t} - e^{3t} - \frac{1}{2}e^{2t}$

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