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1 $\frac{dy}{dt} + 3y = e^{-2x}$

$y' + 3y = e^{-2t}$

$L[y' + (t)] = 5y(3) - y(0)$

$L[y(t)] = y(3)$

$L[e^{-2t}] = \frac{1}{y+2}$

$= Sy(3) - y(0) + 3y(y) = \frac{1}{s+2}$

$Y(s)(s+3) - y(0) = \frac{1}{s+2}$

$Y(s)(s+3-2) = \frac{1}{s+2}$

$Y(s)(s+3) = \frac{1}{s+2} + \frac{2}{1}$

$y(s)(s+3)$

$y(s)(s+3) = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2} = \frac{2s+5}{y+2}$

$y(s) = \frac{2s+5}{s+2} \times \frac{1}{s+3} = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)s+3} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(0+3) + B(s+2)$

at $s = -3$

$2(-3) + 5 = A(-3+3) + B(-3+2)$

$-6+5 = -B$

$-1 = -B$

$B = 1$, at $s = -2$

$2(-2)+5 = A(-2+3)+B(-2+2)$

$-4+5 = A \quad / A=1$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$$

$$= e^{2t} + e^{-3t}$$

2. $3\frac{dy}{dt} - 6y = \sin 2t$

$3y' + 6y = \sin 2t$

$L[y'(t)] = sy(s) - y(0)$

$L[y(t)] = y(t)$

$L[\sin 2t] = \frac{2}{s+2^2}$

$3(sy(t) - y(t) + 6y(t) = \frac{2}{s^2+2^2}$

$= (3s-6)\,y(t) - 3 = \frac{2}{s^2+4}$

$(3s-6)\,r(0) = \frac{2}{s^2+4} + 3 = \frac{2}{(s+2)^2} + \frac{3}{1} = \frac{2+3(s+2)^2}{(s+2)^2}$

$$y(t) = \frac{2+3(s+2)^2}{(s+2)^2} \times \frac{1}{(3s-6)}$$

$$= \frac{2+3(s+2)^2}{(3+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{s+2)} + \frac{C}{3s-6}$$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

at $s = 2$

$3(0)^2 + 12(2) + 14 = A(-2+2)(3(-2)-6) + B(3(-2)-6) + (C-2+2)^2$

$12 - 24 + 14 = -12B$

$2 = -12B$, $B = -1/6$

$3s^2 + 12s + 14 = A(3s^2 - 12) + B(3s - 6) + C(s^2 + 4s + 4)$

$= 3A + C = 3$

$3B + 4C = 12$

$-12A - 6B + 4C = 14$

$3A + C = 3$

$A = \frac{3 - C}{3}$

$-12A - 6B + 4C = 14$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C + 6B + 4C = 14$

$-6B + 8C = 26$

$-6(1/6) + 8C = 26$

$8C = 26 - 1$

$C = \frac{25}{8}$

$A = \frac{3 - 25/8}{3} = \frac{-1/8}{3} = \frac{-1}{24}$

$= \frac{-1/24}{s+2} + \frac{1/6}{(s+2)^2} + \frac{25/8}{3s-1}$

$-1/24\left[\frac{1}{s+2}\right] = 1/6\left[\frac{1}{(s+2)^2}\right] + \frac{25}{8}\left[\frac{1}{3}(s+2)\right]$

$= L^{-1}\left[-1/24\left[\frac{1}{s+2}\right] - 1/6\left[\frac{1}{(s+2)^2}\right] + \frac{25}{24}\left[\frac{1}{s-2}\right]\right]$

$\frac{-1}{24} e^{-2t} - \frac{1}{6} t e^{-2t} + \frac{25}{24} e^{2t}$

3 $\frac{dy}{dt} - 4y = 8$ at $t = 0, y = 2$

$L[y'] = s Y(t) - y(0)$

$L[y] = Y(t)$

$L[e] = \frac{8}{s}$

$s Y(t) - y_0 - 4Y(t) = 8/s$

$(s-4) Y(t) - y(0) = \frac{8}{s}$

$(s-4) Y(t) - 2 = \frac{8}{s}$

$(s-4) Y(t) = \frac{8}{s} + 2$

$Y(t) = \frac{8+2s}{s} \times \frac{1}{s-4}$

$= \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$8 + 2s = A(s-4) + Bs$

at $s = 4$

$8 + 2(4) = A(4-4) + 4B$

$16 = 4B, \quad B = 4$

at $s = 0$

$8 + 2(0) = A(0-1) + B(0)$

$8 = -4A, \quad A = -2$

$= \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}\left[\frac{-2}{s}\right] + L^{-1}\left[\frac{4}{s-4}\right]$

$= -2 + 4e^{4t}$

4 $\frac{d^2y}{dt^2} - \frac{3dy}{dt} + 5y = e^{-2}$

at $t=s$ $y=2$ $y'=-1$

$y'' - 2y' + 5y = e^{2t}$

$L[y''(t)] = s^2Y(t) - s4(0) - y'(0)$ $L[e^{3t}] = \frac{3}{s-2}$

$L[y'(t)] = Y(0)$

$= s^2y(t) - sy(0) - y^2(0) - 2(sy(t) - y(0)) + 5y(t)^2 = \frac{1}{s-2}$

$s^2y(t) - sy(0) - y'(0) - 2sy(t) + 2y(0) + 5y(t) = \frac{1}{s-2}$

$(s^2 - 2s + 5)y(t) - sy(0) - y'(0) + 2Y(t) = \frac{1}{s-2}$

$(s^2 - 2s + 5)y(t) - 2s - 1 + 4 = \frac{1}{s-2}$

$(s^2 - 2s + 5)y(t) - 2s + 3 = \frac{1}{s-2}$

$(s^2 - 2s + 5)y(t) = \frac{1}{s-2} + 2s - 3$

$(s^2 - 2s + 5)y(t) = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$s^2 - 2s + 5)y(t) = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2y^2 - 7s + 7}{s-2}$

$y(t) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{B}{s^2-2} + \frac{C}{s+5}$

$\frac{2s^2 - 7s + 7}{(s-2(s^2 - 2s + 3)} = \frac{A(s^2 - 2s + 5) + Bs + C(s-2)}{(s-2)(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + 2(s + 2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$A + B = 2 \quad \cdots (1)$

$-2A - 2B + C = -7 \quad \cdots (2)$

$8A - 2C = 7 \quad \cdots (3)$

at $s = 2$

$2(s)^2 - 7(2) + 7 = A(x^2 - 2(2) + 5)$

$2 = 5A + 7 = 5A$

$5A = 1$

$A = \frac{1}{5}$

From (1)

$A + B = 2$

$B = 2 - \frac{1}{5}$

$\frac{10 - 1}{5} = \frac{9}{5}$

$5A - 2C = 7$

$5\left(\frac{1}{5}\right) - 2 = 7$

$1 - 2C = 7$

$-2C = 6$

$C = -3$

$= \frac{\frac{1}{5}}{s - 2} + \frac{\frac{9}{5}s - 3}{s^2 - 2s + 5}$

$= \frac{\frac{1}{5}}{(s-2)} + \frac{\frac{9}{5}}{s^2 - 2s + 5} - \frac{3}{s^2 - 2s + 5}$

$\frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left(\frac{s - 1 + 1}{(s-1)^2 + 4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2 + 4}\right)$

$= L^{-1}\left[\frac{\frac{1}{5}}{s-2} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2 + 2^2} + \frac{1 \times \frac{3}{2}}{(s-1)^2 + 2^2}\right] - \frac{3}{2}\left[\frac{2}{(s-1)^2 + 2^2}\right]\right.$

$= L^{-1}\left[\frac{\frac{1}{5}}{s-2} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2 + 2^2} + \frac{1}{2}\left[\frac{2}{(s-1)^2 + 2^2}\right]\right.\right.$

$- \frac{3}{2}\left[\frac{2}{(s-1)^2 + 2^2}\right]$

$= \frac{1}{5} e^{2t} + \frac{9}{5}\left[e^t \cos 2t + \frac{1}{2} e^t \sin 2t\right] - \frac{3}{2}\left[e^t \sin 2t\right]$

5 $\frac{d^2y}{2t^2} = 6\frac{dy}{dt} + 8y = e^{3t}$

at $t=0 \quad y=0 \quad y^t = 2$

$y'' = 6y' + 8y = e^{3t}$

$L[y'(t)] - s^2 y(t) - sy(0) - y(0)$

$L[y^2(t) = sy(t) - y(0)$

$L[y[t] = y(t)$

$L[e^{3t}] = \frac{1}{s-3}$

$= s^2 y(t) - sy(0) = y'(0) - 6(sy(t) - y(0)) + 8y(t) = \frac{1}{s-3}$

$2(s^2 - 6s + 8) Y_t + (-s + 6) y(0) = y'(0) = \frac{1}{s-3}$

$(s^2 - 6s + 8) + (.t) + (-3 + 6) 0 - 2 = \frac{1}{s-3}$

$(s^2 - 6s + 8) y(t) = \frac{1}{s-3} + \frac{2}{1}$

$\frac{1 + 2(s-3)}{s-3} = \frac{1 + 2s - 6}{s-3} = \frac{-5 + 2s}{s-3}$

$y(t) = \frac{s + 2s}{5-3} \times \frac{1}{s^2 - 6s + 8}$

$= \frac{-5 + 2s}{(s-3)(s-4)(s-2)} = \frac{A}{(s-3)} + \frac{B}{s-4} + \frac{C}{s-2}$

$2s - 5 = A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)$

when $s = 4$

$2(4) - 5 = A(4-4)(4-2) + B(4-3)(4-2) + C(4+3)(4-5)$

$3 = 2B$

$B = 3/2$

when $s = 2$

$2(2) - 5 = A(2-4)(2-2) + B(2-3)(2-2) + C(2-3)(2-4)$

$4 - 5 = 2C$

$-1 = 2C$

$C = -1/2$

when $s = 3$

$2(3) - 5 = A(3-4)(3-2) + B(3-3)(3-2) + C(3-3)(3-4)$

$\therefore 1 = -A$

$A = -1$

$= \frac{-1}{s-3} + \frac{3/2}{s-4} + \frac{-1/2}{s-2}$

$= L^{-1}\left[\frac{-1}{s-3}\right] + \left[\frac{3/2}{s-4}\right] - \frac{1}{2}\left[\frac{1}{s-2}\right]$

$= -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$

$= \frac{3}{2}e^{4t} - e^{3t} - \frac{1}{2}e^{2t}$.