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Matric 15/ENG06/022
Dept Mech Engr
Course Eng 281 Assignment 1

1(a) $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4)\sin(\cos x)}{x - \pi/2} \right]$

$$\frac{(x^2 - \pi/4)\sin(\cos x)\,1/\cos x}{(x - \pi/2)\,1/\cos x}$$

$$= \frac{\frac{4x^2 - \pi}{4} \sin(\cos x)\,1/\cos x}{\frac{2x - \pi}{2}\,1/\cos x}$$

$$\lim_{x \to \pi/2} \frac{\frac{(4x^2 - \pi)\sin(\cos x) \cdot 1}{4\cos x}}{\frac{2x - \pi}{2\cos x}}$$

$$\lim_{x \to \pi/2} \frac{\frac{4x^2 \sin(\cos x) - \pi \sin(\cos x)}{4\cos x}}{\lim_{x \to \pi/2} \frac{2x - \pi}{2\cos x}}$$

$$\frac{1/4 \lim_{x \to \pi/2} \frac{4x^2 \sin(\cos x) - \pi \sin(\cos x)}{\cos x}}{1/2 \cdot \lim_{x \to \pi/2} \frac{2x - \pi}{\cos x}}$$

Differentiate (using L'Hopital rule)

$$\frac{1/4 \lim_{x \to \pi/2} \frac{d/dx\,(4x^2 \sin(\cos x) - \pi \sin(\cos x))}{d/dx\,(\cos x)}}{1/2 \lim_{x \to \frac{\pi}{2}} \frac{d/dx\,(2x - \pi)}{d/dx\,(\cos x)}}$$

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Numerator

$$\lim_{x\to\pi/2} \frac{d/dx\,(4x^2\sin(\cos x)) - \pi\sin(\cos x)}{\frac{d/dx(\cos x)}{4}}$$

$$\frac{d/dx(4x^2\sin(\cos x)) + d/dx(-\pi\sin(\cos x))}{\frac{-\sin x}{4}}$$

$$\frac{4(-x^2\sin x\cos(\cos x) + 2\sin(\cos x)x) + \pi\sin x\cos(\cos x)}{-\sin x}$$

$$\frac{-4x^2\sin x\cos(\cos x) + 8x\sin(\cos x) + \pi\sin x\cos(\cos x)}{\sin x}$$

$$= \frac{(4(\pi/2)^2\sin(\frac{\pi}{2})\cos(\cos(\pi/2)) + 8\frac{\pi}{2}\sin(\cos(\frac{\pi}{2})) + \pi\sin(\frac{\pi}{2})\cos(\cos\pi/2)))}{\sin(\frac{\pi}{2})}$$

$$-\frac{(-4\frac{\pi^2}{2^2}(1)\cos(0) + 8\frac{\pi}{2}\sin(0) + \pi\cdot 1\cos(0))}{1}$$

$$-(-4(1)\frac{\pi^2}{2^2} + \pi(1) + 8\frac{\pi}{2}(0))$$

$$-(-4\pi^2/2^2 + \pi)$$

$$-(\frac{-4\pi^2}{4} + \pi)$$

$$-(-\pi^2 + \pi).$$

Denominator

$$\frac{1}{2}\lim_{x\to\pi/2} \frac{2x-\pi}{\cos x}$$

$$= \frac{1}{2}\lim_{x\to\frac{\pi}{2}} \frac{\frac{d}{dx}(2x-\pi)}{d/dx(\cos x)}$$

$$= \frac{1}{2}\,\frac{d/dx\,(2x) + \frac{d}{dx}(-\pi)}{-\sin x}$$

$$= \frac{2\,d/dx\,(x) + 0}{-\sin x}$$

$$= \frac{0 + 2(1)}{-\sin x}$$

$$= \frac{2}{-\sin x}$$

$$= \lim_{x \to \pi/2} -2/\sin x$$

$$(-2) \lim_{x \to \pi/2} \frac{1}{\sin x}$$

$$= -2/1 = -2$$

$$-\frac{(-\pi^2 + \pi)}{4} \quad \frac{2}{-2}$$

$$\frac{-(-\pi^2 + \pi) \cdot 2}{-4 \cdot 2}$$

$$= \frac{(-\pi^2 + \pi)}{4}$$

$$\lim_{x \to \frac{\pi}{2}} -2/\sin x$$

$$= -1.682003$$

b. $\lim_{x \to \frac{\pi}{2}} \ln[\exp \frac{(3x^2 + 2x - 1)}{x + 1}$

soln

$$\ln(\exp) \lim_{x \to \frac{\pi}{2}} \frac{3x^2 + 2x - 1}{x + 1}$$

$$\frac{\lim_{x \to \frac{\pi}{2}} (3x^2 + 2x - 1)}{\lim_{x \to \frac{\pi}{2}} (x + 1)}$$

$$\frac{(3(\frac{\pi}{2})^2 + 2(\frac{\pi}{2}) - 1)}{(\frac{\pi}{2}) + 1}$$

$$\frac{\frac{3\pi^2}{4} + \frac{2\pi}{2} + 1}{\frac{\pi}{2} + 1}$$

$$\left(\frac{3\pi^2}{4} + \pi - 1\right) \cdot \frac{2}{\pi + 2}$$

$$= \frac{6\pi^2 + 8\pi - 8}{4\pi + 8}$$

$$= \frac{3\pi^2 + 4\pi - 4}{2\pi + 4}$$

$$= 3.712399$$

c. $\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$

soln

$$\lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$$

$$= \cos\left[\sin^{-1}\frac{(2+\sqrt{3}-2)}{(2+\sqrt{3}-\sqrt{3})}\right]$$

$$= \cos\left[\sin^{-1}(\sqrt{3}/2)\right].$$

$$= \cos(60) = 1/2.$$

$\therefore \lim_{x \to 2+\sqrt{3}} \cos\left[\sin^{-1}\frac{(x-2)}{(x-\sqrt{3})}\right]$ is $1/2$

d. $\lim_{x \to 4}\left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right].$

soln

$$\lim_{x \to 4}\left[\frac{(x-4)(x-4)}{(x-1)(x-4)}\right]$$

$$\Rightarrow \lim_{x \to 4}\left[\frac{(x-4)}{(x-1)}\right] = 0/3 = 0$$

Elae Paul

2a. Determine which is convergent

$$\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6}$$

Solution

$$\frac{U_{n+1}}{U_n}$$

$$U_n = \frac{2}{(n+1)(n+2)}$$

$$U_{n+1} = \frac{2}{(n+2)(n+3)}$$

$$\frac{U_{n+1}}{U_n} = \frac{2}{(n+2)(n+3)} \div \frac{2}{(n+1)(n+2)}$$

$$= \frac{\cancel{2}}{(n+2)(n+3)} \times \frac{(n+1)(n+2)}{\cancel{2}}$$

$$= \frac{1}{(n+1)(n+3)} \times \frac{(n+1)(n+1)}{1}$$

$$= \frac{\cancel{(n+1)}(n+1)}{\cancel{(n+1)}(n+3)}$$

$$= \frac{n+1}{n+3} = \frac{1+1/n}{1+3/n}$$

$$= \frac{1+0}{1+0} = 1$$

It is Convergent

b $$\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \dots$$

soln

$$\frac{U_{n+1}}{U_n} \quad U_{n+1} = \frac{2}{(n+1)^2} \quad U_n = \frac{2}{n^2}$$

$\frac{u_{n+1}}{u_n}$

$$\frac{2}{(n+1)^2} \div \frac{2}{n^2}$$

$$\frac{2}{(n+1)^2} \times \frac{n^2}{2}$$

$$\frac{2}{(n+1)(n+1)} \times \frac{n^2}{2}$$

$$\frac{1}{(n+1)^2} \times n^2 = \frac{n^2}{(n+1)^2}$$

$$= \frac{n^2}{n^2+2n+2}$$

$$= \frac{1}{1+2/n+2/n^2}$$

$$= \frac{1}{1+0+0} = 1$$

It is convergent

-). $u_n = \frac{1+2n^2}{1+n^2}$

Divide by n^2

$$= \frac{\frac{1}{n^2}+2}{1/n^2+1}$$

$$= \frac{0+2}{0+1} = 2/1$$

$u_n \neq 0$ and $P > 1$

$\therefore$ It is divergent

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3) Find the range of value of x for which the series below is absolutely convergent

$$\frac{x}{27} + \frac{x^2}{125} + \dots + \frac{x^n}{(2n+1)^3}$$

soln

$$u_n = \frac{x^n}{(2n+1)^3}$$

$$u_{n+1} = \frac{x^{n+1}}{(2n+1+1)^3} = \frac{x^{n+1}}{(2n+2)^3}$$

$$\frac{u_{n+1}}{u_n} = \frac{x^{n+1}}{(2n+2)^3} \times \frac{(2n+1)^3}{x^n}$$

$$= x \times \frac{(2n+1)^3}{(2n+2)^3}$$

$$\frac{8n^3 + 12n^2 + 6n + 1}{8n^3 + 32n^2 + 32n + 8}$$

$$= \frac{8 + 12/n + 6/n^2 + 1/n^3}{8 + 32/n + 32/n^2 + 8/n^3}$$

$$= \frac{8}{8} = 1$$

∴ The series is convergent for all values of x

4) Evaluate Using L'Hopital's Rule

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right)$$

soln

$$\lim_{x \to 0} \left(\frac{\sin x - \cos x}{x^3} \right) \quad \left\{ \frac{\sin 0 - \cos 0}{0^3} = \text{undefined} \right.$$

Differentiate

$$\frac{d}{dx} = \frac{\cos x + \sin x}{3x^2}$$

$$\frac{\cos 0 + \sin 0}{3 \times 0} = \text{undefined.}$$

differentiate again

$$\frac{d}{dx} = \frac{-\sin x + \cos x}{6x}$$

$$\frac{-\sin 0 + \cos 0}{6 \times 0} = \text{undefined}$$

differentiate

$$\frac{d}{dx} = \frac{-\cos x - \sin x}{6}$$

$$\frac{-\cos 0 - \sin 0}{6} = \frac{-1}{6}$$