Name Elue Paul Ifeanyichukwu
Matric 15/ENG06/022
Course ENG 281
Dept Mechanical Engineering
Assignment 2

The parametric equations of a curve are

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

Determine

a) an expression for R

b) expressions for the coordinates (h, k) of the centre of curvature

a)

$x = \cos t + t\sin t$

$y = \sin t - t\cos t$

$\frac{dx}{dt} = ?$

u v

$t\sin t + \cos t$

$= uv + (-\sin t)$

$u = t \quad du = 1$

$v = \sin t \quad dv = \cos t$

$v\frac{du}{dx} + u\frac{dv}{dx}$

$\sin t(1) + t(\cos t)$

$\frac{dx}{dt} = \sin t + t\cos t - \sin t$

$\frac{dy}{dt} = \cos t - (-t\sin t + \cos t)$

$= \cos t + t\sin t - \cos t = t\sin t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = t\sin t \times \frac{1}{t\cos t} = \frac{t\sin t}{t\cos t}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx} \qquad \left(\frac{d}{dx}\left(\frac{dy}{dx}\right) \div \frac{dx}{dt}\right)$$

R

with quotient rule

$$\frac{d^2y}{dx^2} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = t\sin t \quad du = t\cos t + \sin t$

$v = t\cos t \quad dv = -t\sin t + \cos t$

$$= \frac{(t\cos t)\cdot(t\cos t + \sin t) - (t\sin t)\cdot(-t\sin t + \cos t)}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2\cos^2 t + t\cos t\sin t + t^2\sin^2 t - t\sin t\cos t}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2\cos^2 t + t^2\sin^2 t}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2(\cos^2 t + \sin^2 t)}{(t\cos t)^2} \times \frac{1}{t\cos t}$$

$$= \frac{t^2}{(t\cos t)^2} \times \frac{1}{t\cos t} = \frac{t^2}{t^2\cos^2 t} \times \frac{1}{t\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{t\cos^3 t}$$

b

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$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left(1 + \left(\frac{t\sin t}{t\cos t}\right)^2\right)^{3/2}}{1/t\cos^3 t}$$

$$= \frac{\left(\frac{1}{1} + \frac{t^2\sin^2 t}{t^2\cos^2 t}\right)^{3/2}}{1/t\cos^3 t}$$

$$= \left(\frac{t^2\cos^2 t + t^2\sin^2 t}{t^2\cos^2 t}\right)^{3/2} * t\cos^2 t$$

$$= \left(\frac{t^2(\cos^2 t + \sin^2 t)}{t^2\cos^2 t}\right)^{3/2} = \left(\frac{t^2}{t^2\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

$$= \left(\frac{1}{\cos^2 t}\right)^{3/2} \times t\cos^3 t$$

$$= \left(\frac{1}{\cos^2 t}\right)^{1/2 \times 3} \times t\cos^3 t$$

$$= \left(\frac{\sqrt{1}}{\sqrt{\cos^2 t}}\right)^3 \times t\cos^3 t$$

$$R = t$$

b

$h = x_1 - R\sin t$

$k = y_1 + R\cos t$

$x_1 = \cos t + t\sin t$

$y_1 = \sin t - t\cos t$

$h = \cos t + t\sin t - t\sin t = \cos t$

$k = \sin t - t\cos t + t\cos t = \sin t$

$(h, k) = (\cos t, \sin t)$

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