4

Name: Ayeni Olatunbosun.
15/ENG07/007
Petroleum Engineering
ENG 381

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

General Equation

$$(uv)^n = u^n v^0 + nu^{n-1}v^{1} + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!} + \ldots\ldots$$

for $(1-x^2)y''$ let $u = y''$, $u' = y'''$, $u'' = y^{iv}$; $(uv)^n = y^{n+2}$

$v = v^0 = 1-x^2$, $v' = -2x$, $v'' = -2$, $v''' = 0$

for xy'

$u = y'$, $u' = y''$, $u'' = y'''$, $u^n = y^{n+1}$

$v = v^0 = x$, $v' = 1$, $v'' = 0$

$(uv)_2^n = x \cdot y^{n+1} + n \cdot y^n \cdot 1$

$= xy^{n+1} + ny^n$

for $y = y^n = (uv)_3^n$

$(uv)^n = (uv)_1^n + (uv)_2^n + (uv)_3^n$

$(uv)^n = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2(xy^{n+1} + ny^n)$

$0 = (1-x^2)y^{n+2} - 2xny^{n+1} - (n^2-n)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

If $x = 0$

$0 = y^{n+2} - (n^2-n)y^n - 2ny^n + 2y^n$

$y^{n+2} = (n^2 - n + 2n - 2)y^n$

$y^{n+2} = (n^2 + n - 2)y^n$

for $n = 1$

$y^3 = y''' = (1^2 + 2 - 2)y' = 0$

for $n = 2$

$y^4 = y^{iv} = (2^2 + 2 - 2)y'' = 4y''$

for $n = 3$

$y^5 = y^v = (3^2 + 3 - 2)y''' = 10y''' = 0$

for $n = 4$

$y^6 = y^{vi} = (4^2 + 4 - 2)y^{iv} = 18y^{iv} = 18(4y'')$

for $n = 5$

$y^7 = y^{vii} = (5^2 + 5 - 2)y^v = 28y^v = 0$

for $n = 6$

$y^8 = y^{viii} = (6^2 + 6 - 2)y^{vi} = 40y^{vi} = [40 \times 18 \times 4]\, y''$

$$y = 1 + f(x) + y' f'(x) + \frac{y'' f''(x)}{2!} + \cdots$$

$$y = 1 + y^0 + y' + \frac{y''}{2!} + \frac{4y''}{4!} + \frac{18y''(4)}{6!} + \frac{20((8\times4))y''}{8!}$$

$$y = 1 + y^0 + y' + y''\left(\tfrac{1}{2} + \tfrac{1}{6} + \tfrac{1}{10} + \tfrac{1}{14}\right)$$

2i) $L(3e^{-4t} - 5e^{4t}) = \frac{3}{s+4} - \frac{5}{s-4}$

ii) $L(\sin 4t + \cos 4t) = \frac{4}{s^2+4^2} + \frac{5}{s^2+4^2} = \frac{4+5}{s^2+16}$

iii) $L(t^3 + 2t^2 - t + 4) = \frac{3!}{s^{3+1}} + \frac{2(2!)}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$

$$= \frac{1}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv) $L(e^{-2t}\cos 5t) = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{s^2+4s+29}$

v) $L(t\sin 3t) = (-1)^n \cdot \frac{d^n}{dx^n}(f(t)) = -1\frac{d}{dx}\left[1\ \frac{3}{s^2+3^2}\right]$ $du = 0$ $dv = 2s$

$$= \left(\frac{-6s}{(s^2+9)^2}\right) = \frac{6s}{(s^2+9)^2}$$

vi) $L(e^{4t}\cos 2t) = \frac{s}{(s-4)^2+2^2} = \frac{s}{s^2-8s+20}$

viii) $L(t\sin 2t) = -1 \cdot \frac{d}{dx}\left[\frac{2}{s^2+2^2}\right]$ $du = 0$ $dv = 2s$

$$= -1\left[\frac{-4s}{(s^2+2^2)^2}\right] \quad du = 0 \quad dv = 2s \quad = \frac{4s}{(s^2+4)^2}$$

ix) $t^3 + 4t^2 + 5 = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+3}} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{6}$

x) $e^{3t}(t^2+4) = t^2e^{3t} + 4e^{3t} = \frac{2!}{(s-3)^{1+1}} + \frac{4}{s-3}$

$$= \frac{2}{(s-3)^2} + \frac{4}{s-3}$$

x) $t^2 \cos t = (-1)^2 \cdot \frac{d^2}{dx^2}\left[\frac{s}{s^2+1}\right] = \frac{d}{dx}\left[\frac{d}{dx}\left(\frac{s}{s^2+1}\right)\right]$ $du = 1$, $dv = 2s$

$$= \frac{d}{dx}\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right] = \frac{d}{dx}\left[\frac{1-s^2}{(s^2+1)^2}\right]$$ $du = -2s$, $dv = 4s^3 + 4s$

$$= \left[\frac{-2s^5 - 4s^3 - 2s - 4s + 4s^5}{[(s^2+1)^2]^2}\right] = \left[\frac{2s^5 - 4s^3 - 6s}{(s^2+1)^4}\right]$$

xiii) $\frac{\sinh 2t}{t} = \frac{1}{2}\ln(s^2-4) - \ln s$

3i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

at $s = 3$

$3-5 = A(3-4)$

$A = 2$

at $s = 4$ $\therefore 4-5 = B(4-3)$

$\therefore L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right] = 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$

$2s-6 = A(s-4) + B(s-2)$

at $s = 2$

$2(2) - 6 = A(2-4) + B(2-2)$

$4 - 6 = A(-2)$.

$-2 = -2A \Rightarrow A = 1$

at $s = 4$

$2(4) - 6 = A(4-4) + B(4-2)$

$8 - 6 = B(2)$

$2 = 2B \Rightarrow B = 1$

$L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right] = e^{2t} + e^{4t}$.