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Course: ENG 351

(1) $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$y'' + 4y' + 5y = 6\sin\theta$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

$k^2 + 4k + (2)^2 = -5 + (2)^2$

$(k+2)^2 = -1$

$k + 2 = \pm\sqrt{-1}$

$k + 2 = \pm i$

$k_1 = -2 + i$ and $k_2 = -2 - i$

$y_h = c_1 e^{(-2+i)\theta} + c_2 e^{(-2-i)\theta}$

$y_h = c_1 e^{-2\theta+i\theta} + c_2 e^{-2\theta-i\theta}$

$y_h = c_1 e^{-2\theta} e^{i\theta} + c_2 e^{-2\theta} e^{i\theta}$

$y_h = e^{-2\theta}\left[c_1 e^{i\theta} + c_2 e^{-i\theta}\right]$

$y_p = A\cos\theta + B\sin\theta$

$y'_p = -A\sin\theta + B\cos\theta$

$y''_p = -A\cos\theta - B\sin\theta$

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$4A\cos\theta + 4B\sin\theta - 4A\sin\theta + 4B\cos\theta = 6\sin\theta$

$(4A + 4B)\sin\theta + (4A + 4B)\cos\theta = 6\sin\theta$

$-4A + 4B = 6$

$4A + 4B = 0$

$8B = 6$

$B = \frac{6}{8} = 3/4$

$4A = -4B$

$A = -B \quad A = -3/4$

$$\therefore y_p = \frac{-3}{4}\cos\theta + \frac{3}{4}\sin\theta$$

$$y = y_h + y_p$$

$$y = e^{2\theta}[A\cos\theta + B\sin\theta] + \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$$

ii.) Steady state equation

$$y_p' = 0$$

$$y_p' = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$$

$$\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$$

$$\frac{3}{4}\cos\theta = -\frac{3}{4}\sin\theta = 0$$

$$\cos\theta = -\sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}$$

$$\tan\theta = -1$$

If $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

$$EIm^2 = 0$$

$$m^2 = 0$$

$$m = \pm\sqrt{0}$$

$$m = \pm 0$$

$$y = e^{0x}[A + Bx]$$

$$y = A + Bx$$

$$y_p = y = Fx^2 + Gx^3 + Hx^4$$

$$\frac{dy}{dx} = 2Fx + 18x^2 + 4Hx^3$$

$$\frac{d^2y}{dx} = 2F + 36x + 12Hx^2$$

$$EI\{2F + 36x + 12Hx^2\} = w/2(L-x)^2$$

$$2FEI + 36EIx + 12HEIx^2 = \frac{w}{2}(L-x)^2$$

$$4FEI + xEIx + 2\phi H = w\frac{(L^2 - 2Lx + x^2)}{2}$$

$$2\phi H + 2EI = w$$

$$H = \frac{w}{24EI}$$

$$12EI = -2wL$$

$$\theta = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$$

$$4FEI = wL^2$$

$$F = \frac{wL^2}{4EI}$$

$$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$$

$$= \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$$

$$= \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$$

$$= y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0$, $x = 0$, $dy/dx = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left(6L^2(0) - 4L(0) + 0\right)$$

$$dy/dx = B + w/24EI\left[12L^2x - 12Lx^2 + 4x^3\right]$$

$$A = 0$$

$$0 = \frac{dy}{dx} = B + \frac{w}{24EI}\left[12L(0) - 12(0) + 4L(0)\right]$$

$$B = 0$$

$$y_p = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y_p = \frac{wx^2}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

$$y_p = \frac{wx^2}{24EI}\left[x^2 - 4Lx + 6L^2\right]$$

when $x = L$

$$y_1 = \frac{wL^2}{24EI}\left[L^2 - 4L^2 + 6L^2\right]$$

$$y_2 = \frac{wL^2}{24EI}\left[3L^2\right]$$

$$y = \frac{wL^4}{8EI}$$