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ENG381

1) If $y = e^{x^2} + x$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y' = (2x+1)e^{x^2+x}$

$y'(2x+1) = (2x+1)e^{x^2+x}(2x+1)$

$= (2x+1)^2 e^{x^2+x}$

$2y = 2 \cdot e^{x^2+x}$

$y'(2x+1) + 2y = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$2e^{x^2+x} + (2x+1)^2 e^{x^2+x} = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$w = y''$

$w^n = y^{n+2}$

$P = y'(2x+1)$

$v = 2x+1$

$v' = 2 \quad u = y'$

$v'' = 0 \quad u^n = y^{n+1}$

$P^n = y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2$

$S = 2y$

$S^n = 2y^n$

$w^n = P^n + S^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n-1)y^n$

2) Using leibnitz theorem, given that $y = x^3 e^{4x}$ determine y_5

$v = x^3$ $\quad u = e^{4x}$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v'' = 6x$ $\quad u'' = 16e^{4x} = 4.4 e^{4x}$

$v''' = 6$ $\quad u''' = 4.4.4 e^{4x}$

$v^{iv} = 0$ $\quad u^n = 4^n e^{4x}$

$y^n = 4^n e^{4x} . x^3 + n . 4^{n-1} e^{4x} . 3x^2 + \frac{n(n-1)}{2} 4^{n-2} 6x + \frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} . 6$

$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$

$y^5 = 4^5 e^{4x} x^3 + 4 e^{4x} 3x^2 + 4^3 e^{4x} 3x + 5.4.3.4^2 . e^{4x}$

3) $x^2 y'' + xy' + y = 0$ Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$.

Let $w = x^2 y''$

$v = x^2$ $\quad u = y''$

$v' = 2x$ $\quad u' = y'''$

$v'' = 2$ $\quad u'' = y^{(iv)}$

$v''' = 0$ $\quad u^n = y^{n+2}$

$w^n = y^{n+2} x^2 + n . y^{n+1} 2x + \frac{n(n-1)}{2} . y^n . 2$

$w^n = x^2 y^{n+2} + n 2x y^{n+1} + n(n-1) y^n$

Let $P = xy'$

$v = x$. $u = y'$

$v' = 1$ $\quad u'' = y''$

$v'' = 0$ $\quad u^n = y^{n+1}$

$P^n = y^{n+1} . x + n . y^n . 1$

$P^n = x y^{n+1} + n y^n$

$S = y$

$S^n = y^n$

$y^{n+2} x^2 + n . 2x y^{n+1} + n(n-1) y^n + y^{n+1} x + n y^n + y^n$

$y^{n+2}x^2 + (n-2x\ y^{n+1} + xy^{n+1}) + [n(n-1)y^n + ny^n + y^n] = 0$

$x^2y^{n+2} (2n+1)xy^{n+1} + (n(n-1) + 1 + n)y^n = 0$

$x^2y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$