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Assignment

i) $\frac{dy}{dt} + 3y = e^{-2t}$ at $t = 0$ and $y = 2$

$y' + 3y = e^{-2t}$

$Sy(s) - y(s) + 3y(s) = \frac{1}{s+2}$

$Sy(s) - 2 + 3y(s) = 1/s+2$

$y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1}$

$y(s)[s+3] = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A: \left.\frac{1+2s+4}{s+3}\right|_{s=-2}$

$\Rightarrow \frac{1+2(-2)+4}{-2+3} = 1$

$B: \left.\frac{1+2s+4}{s+2}\right|_{s=-3}$

$\Rightarrow \frac{1+2(-3)+4}{(-3)+2} = 1$

$y(s) = 4/s+2 + \frac{1}{s+3}$

$= e^{-2t} = e^{-3t}$

2) $3\,dy/dt - 6y = \sin 2t$

$3y' - 6y = \sin 2t$ at $t=0$ $y=1$

$3[s\,y(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3s\,y(s) - 3(y(0)) - 6y(s) = \frac{2}{s^2+4}$

$y(s)[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$

$y(s)[3s-6] = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+4)}$

$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{(3s-6)}$

C: $\left.\frac{3s^2+14}{s^2+4}\right|_{s=2}$

$\frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$

$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$

$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

$3 = 3B + C$

$3 = 3B + 13/4 = 3B = -1/4$ $B = -1/12$

$3A - 6B = 0$

$3A = 6B$

$3A = 6 \times -1/12 =$

$= A = -1/6$

$y(s) = \frac{-1/6 + (-1/12)s}{s^2+4} + \frac{13/4}{3s-6}$

$= \frac{-1/6}{s^2+4} - \frac{1/12\,s}{s^2+4} + \frac{13/4}{3s-6}$

$= -\frac{1}{6} \cdot \frac{1}{s^2+2^2} - \frac{1}{12} \cdot \frac{s}{s^2+2^2} + \frac{13}{4} \cdot \frac{1}{3(s-2)}$

$= -\frac{1}{6} \cdot \frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{14}\left[\frac{1}{s-2}\right]$

$$y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$$

$$y(t) = \frac{1}{12}\left[13e^{2t} - \cos 2t - \sin 2t\right]$$

3) $\frac{dy}{dt} - 4y = 8$

$y' - 4y = 8$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = 8/s$

$y(s)[s-4] = 8/s + \frac{2}{1} = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A: \left.\frac{8+2s}{s-4}\right|_{s=0}$

$= \frac{8}{-4} = -2$

$B: \left.\frac{8+2s}{s}\right|_{s=4}$

$\frac{8+2(4)}{4} = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

at $t=0$ $y=2$ $y'=1$

$y^{(2)} - 2y^{(1)} + 5y = e^{2t}$

$= \left(s^2y(s) - sy(0) - y'(0)\right) - 2\left(sy(s) - y(0)\right) + 5y(s) = \frac{1}{s-2}$

$= s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$= y(s)\left[s^2 - 2s + 5\right] = \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1} = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$= y(s)\left[s^2 - 2s + 5\right] = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2s^2 - 7s + 7}{s-2}$

$= y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$

$A| \left.\frac{2s^2 - 7s + 7}{s^2 - 2s + 5}\right|_{s=2}$

$A: \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$

$= 2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$= 2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2 = A + B$

$2 = \frac{1}{5} + B \therefore B = \frac{9}{5}$

$7 = 5A - 2C$

$7 = 5\left(\frac{1}{5}\right) - 2C$

$7 - 1 = -2C$

$C = -3$

$y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$

$y(s) = \frac{1}{5}\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+4}$

$y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}\cdot\frac{2}{2}$

$$y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2+2^2} - 2\cdot\frac{2}{(s+1)^2+2^2}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 2t - 2e^{-t}\sin 2t$$

$$y(t) = \frac{1}{5}\left[e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t\right]$$

$$y(t) = \frac{1}{5}\left[e^{2t} + e^{-t}(9\cos 2t - 10\sin 2t)\right]$$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$ $y'=2$

Solution

$$y^{(2)} - 6y^{(1)} + 8y = e^{3t}$$

$$s^2 y(s) - sy(0) - y'(0) - 6[sy(s) - y(0)] + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - 2 - 6sy(s) + 8y(s) = 1/s-3$$

$$y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + 2 = \frac{1+2(s-3)}{s-3}$$

$$\Rightarrow \frac{1+2s-6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} + \frac{C}{s-4}$$

$$A: \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3}$$

$$A: \frac{2(3)-5}{(3-2)(3-4)} = -1$$

$$B: \left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = -1/2$$

$$C: \left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$$

$$y(s) = -1/s-3 - 1/2\cdot 1/s-2 + 3/2\cdot\frac{1}{s-4}$$

$$y(t) = -e^{3t} - 1/2\,e^{2t} + 3/2\,e^{4t}$$

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