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Assignment V

1) $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t=0$, $y=2$

Soln

$L\{\frac{dy}{dt}\} = SY_{(s)} - y_{(0)}$

$L\{3y\} = 3Y_{(s)}$

$L\{e^{-2t}\} = \frac{1}{S+2}$

$SY_{(s)} - y_{(0)} + 3Y_{(s)} = \frac{1}{S+2}$

at $t=0$, $y=2$

$SY_{(s)} - 2 + 3Y_{(s)} = \frac{1}{S+2}$

$SY_{(s)} + 3Y_{(s)} = \frac{1}{S+2} + 2$

$SY_{(s)} + 3Y_{(s)} = \frac{1+2S+4}{S+2} = \frac{2S+5}{S+2}$

$Y_{(s)}[S+3] = \frac{2S+5}{S+2}$

$Y_{(s)} = \frac{2S+5}{S+2} \times \frac{1}{(S+3)} = \frac{2S+5}{(S+2)(S+3)}$

$L^{-1}\{Y_{(s)}\} = y_{(t)}$

$\frac{2S+5}{(S+2)(S+3)} = \frac{A}{S+2} + \frac{B}{S+3}$

$2S+5 = A(S+3) + B(S+2)$

at $S=-2$; $-1 = A$, $A=-1$

at $S=-3$; $-3 = -B$, $B=3$

$\frac{2S+5}{(S+2)(S+3)} = \frac{3}{S+3} - \frac{1}{S+2}$

$L^{-1}\left\{\frac{3}{S+3} - \frac{1}{S+2}\right\} = 3L^{-1}\left\{\frac{1}{S+3}\right\} - L^{-1}\left\{\frac{1}{S+2}\right\}$

$y_{(t)} = 3e^{-3t} - e^{-2t}$ $\therefore y = 3e^{-3t} - e^{-2t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ given that $t=0$, $y=1$

$\mathcal{L}\left\{3\frac{dy}{dt}\right\} = 3\mathcal{L}\left\{\frac{dy}{dt}\right\} = 3\left(sY_{(s)} - y_{(0)}\right)$

$= 3sY_{(s)} - 3y_{(0)}$

$\mathcal{L}\{-6y\} = -6Y_{(s)}$

$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$

$3sY_{(s)} - 3y_{(0)} - 6Y_{(s)} = \frac{2}{s^2+4}$

at $t=0$, $y=1$

$3sY_{(s)} - 3(1) - 6Y_{(s)} = \frac{2}{s^2+4}$

$3sY_{(s)} - 6Y_{(s)} = \frac{2}{s^2+4} + 3$

$Y_{(s)}[3s-6] = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y_{(s)} = \frac{3s^2+14}{s^2+4} \times \frac{1}{[3s-6]} = \frac{3s^2+14}{(s^2+4)(3s-6)}$

$\mathcal{L}^{-1}\{Y_{(s)}\} = y_{(t)}$

$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$

$3s^2+14 = As+B(3s-6) + C(s^2+4)$

at $s=2$; $26 = 8C$; $C = 13/4$

at $s=0$; $14 = -6B + 4C$

$14 = -6B + 4(13/4)$

$14 = -6B + 13$; $-6B = 1$

$B = -1/6$

at $s=1$; $17 = A - 3B + 5C$

$17 = A - 3(1/6) + 5(13/4)$

$17 = A - 1/2 + 65/4$

$A = 17 + 1/2 - 65/4$

$A = 5/4$

$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{5/4 s - 1/6}{s^2+4} + \frac{13}{4(3s-6)} = \frac{15s-2}{12(s^2+4)} + \frac{13}{4(3s-6)}$

$$L^{-1}\left\{\frac{15s-2}{12(s^2+4)} + \frac{13}{4(3s-6)}\right\} = \frac{15}{12}L^{-1}\left\{\frac{s-2}{s^2+4}\right\} + \frac{13}{4}L^{-1}\left\{\frac{1}{3s-6}\right\}$$

$$= \frac{15}{12}L^{-1}\left\{\frac{s-2}{s^2+4}\right\} + \frac{13}{12}L^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= \frac{15}{12}L^{-1}\left\{\frac{s}{s^2+4}\right\} - \frac{15}{12}L^{-1}\left\{\frac{2}{s^2+4}\right\} + \frac{13}{12}L^{-1}\left\{\frac{1}{s-2}\right\}$$

$$y = \frac{15}{12}\cos 2t - \frac{15}{12}\sin 2t + \frac{13}{12}e^{2t}$$

$$= \frac{15}{12}(\cos 2t - \sin 2t) + \frac{13}{12}e^{2t}$$

3) $\frac{dy}{dt} - 4y = 8$ given that at $t=0$, $y=2$

Sol^n

$$L\left\{\frac{dy}{dt}\right\} = sY_{(s)} - y_{(0)}$$

$$L\{-4y\} = -4Y_{(s)}$$

$$L\{8\} = \frac{8}{s}$$

$$sY_{(s)} - y_{(0)} - 4Y_{(s)} = \frac{8}{s}$$

at $t=0$, $y=2$

$$sY_{(s)} - 2 - 4Y_{(s)} = \frac{8}{s}$$

$$sY_{(s)} - 4Y_{(s)} = \frac{8}{s} + 2$$

$$Y_{(s)}[s-4] = \frac{8+2s}{s}$$

$$Y_{(s)} = \frac{8+2s}{s} \times \frac{1}{[s-4]} = \frac{8+2s}{(s)(s-4)}$$

$$L^{-1}\{Y_{(s)}\} = y_{(t)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$8+2s = A(s-4) + B(s)$$

at $s=4$; $16 = 4B$

$B = 4$

at $s=0$; $8 = -4A$, $A = -2$

$$\frac{8+2s}{s(s-4)} = \frac{4}{s-4} - \frac{2}{s}$$

$$L^{-1}\left\{\frac{4}{s-4} - \frac{2}{s}\right\} = 4L^{-1}\left\{\frac{1}{s-4}\right\} - L^{-1}\left\{\frac{2}{s}\right\}$$

$$y = 4e^{4t} - 2$$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0$, $y=2$, $y'=1$

$L\{\frac{d^2y}{dt^2}\} = S^2Y_{(s)} - SY_{(0)} - y'_{(0)}$

$L\{-2\frac{dy}{dt}\} = -2(SY_{(s)} - y_{(0)}) = -2SY_{(s)} + 2y_{(0)}$

$L\{5y\} = 5Y_{(s)}$

$L\{e^{2t}\} = \frac{1}{s-2}$

$S^2Y_{(s)} - Sy_{(0)} - y'_{(0)} - 2SY_{(s)} + 2y_{(0)} + 5Y_{(s)} = \frac{1}{s-2}$

at $t=0$, $y=2$, $y'=1$

$S^2Y_{(s)} - 2(s) - 1 - 2SY_{(s)} + 2(2) + 5Y_{(s)} = \frac{1}{s-2}$

$S^2Y_{(s)} - 2S - 1 - 2Y_{(s)} + 4 + 5Y_{(s)} = \frac{1}{s-2}$

$S^2Y_{(s)} + 3Y_{(s)} - 2S + 3 = \frac{1}{s-2}$

$Y_{(s)}(s^2+3) = \frac{1}{s-2} + 2s - 3$

$Y_{(s)}(s^2+3) = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2s^2 - 7s + 7}{s-2}$

$Y_{(s)} = \frac{2s^2 - 7s + 7}{(s-2)(s^2+3)}$ ~~$= \frac{A}{s-2} + \frac{B}{s^2+3}$~~

$L^{-1}\{Y_{(s)}\} = y_{(t)}$

~~$\frac{2s^2-7s+7}{(s-2)(s^2+3)} = \frac{A}{s-2} +$~~

$\frac{2s^2 - 7s + 7}{(s-2)(s^2+3)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+3}$

$2s^2 - 7s + 7 = A(s^2+3) + (Bs+C)(s-2)$

at $s=2$; $1 = 7A$, $A = 1/7$

at $s=0$; $7 = 3A - 2C$

$7 = 3(1/7) - 2C$; $7 - 3/7 = -2C$

$-2C = 46/7$, $C = \frac{-46}{14}$

~~at $s=1$;~~

at $s=1$; $2 = 4A + (B+C)(-1)$

$2 = 4A - B - C$

$2 = 4(1/7) - B + \frac{46}{14}$, $B = 4/7 + \frac{46}{14} - 2$

$B = 13/7$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2+3)} = \frac{1}{7(s-2)} + \frac{\frac{13}{7}s - \frac{46}{14}}{s^2+3} = \frac{1}{7}\left(\frac{1}{s-2}\right) + \frac{26s - 46}{14(s^2+3)}$

$L^{-1}\{\frac{1}{7(s-2)} + \frac{26s - 46}{14(s^2+3)}\} = 1/7\, L^{-1}\{1/(s-2)\} + 1/14\, L^{-1}\{\frac{26s-46}{s^2+3}\}$

$y = 1/7\, e^{2t} + 1/14 \cos\sqrt{3}t$

5)

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0, y=0, y'=2$

$L\{\frac{d^2y}{dt^2}\} = S^2Y_{(s)} - SY_{(0)} - y'_{(0)}$

$L\{-6\frac{dy}{dt}\} = -6[SY_{(s)} - y_{(0)}] = -6SY_{(s)} + 6y_{(0)}$

$L\{8y\} = 8Y_{(s)}$

$L\{e^{3t}\} = \frac{1}{s-3}$

$S^2Y_{(s)} - SY_{(0)} - y'_{(0)} - 6SY_{(s)} + 6y_{(0)} + 8Y_{(s)} = \frac{1}{s-3}$

at $t=0, y=0, y'=2$

$S^2Y_{(s)} - s(0) - 2 - 6SY_{(s)} + 6(0) + 8Y_{(s)} = \frac{1}{s-3}$

$S^2Y_{(s)} - 6SY_{(s)} + 8Y_{(s)} = \frac{1}{s-3} + 2$

$Y_{(s)}(s^2 - 6s + 8) = \frac{1+2s-6}{s-3} = \frac{2s-5}{s-3}$

$Y_{(s)} = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)}$

$L^{-1}\{Y_{(s)}\} = y_{(t)}$

$\frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

at $s=3$; $1 = -A$, $A = -1$

at $s=4$; $3 = 2C$, $C = \frac{3}{2}$

at $s=2$; $-1 = 2B$, $B = -\frac{1}{2}$

$\frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}$

$L^{-1}\{\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}\} = -L^{-1}\{\frac{1}{s-3}\} - \frac{1}{2}L^{-1}\{\frac{1}{s-2}\} + \frac{3}{2}L^{-1}\{\frac{1}{s-4}\}$

$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$

$\therefore y = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$