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PETROLEUM ENGINEERING
ENG 381

1 $\frac{dy}{dt} + 3y = e^{-3t}$

$y(t) + 3y(t) = e^{-2t}$

$L[y'(t)] = sY(s) - Y(s)$

$L[y(t)] = Y(s)$, $L^{-1}[e^{-2t}] = \frac{1}{s+2}$

$sY(s) - Y(s) + 3Y(s) = \frac{1}{s+2}$

$[s-3]Y(s) - 2 = \frac{1}{s+2}$

$(s-3)Y(s) - 2 \quad \frac{1}{s+2}$

$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2(-2)+5 = A(-2-3) \Rightarrow A = \frac{1}{5}$

$2(3)+5 = B(3+2) \Rightarrow B = \frac{11}{5}$

$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5}e^{-2t} + \frac{11}{5}e^{3t}$

ii $3\frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(t)$

$L[y'(t)] = sY(s) - Y(0)$

$L[y(t)] = Y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3sY(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$

$3s^2+14 = A(s^2+4) + B(3s-6)$

$4A - 6B = 14$

$-6B = 14-12$

$B = -1/3$

$Y(s) = \frac{3}{(3)(s-2)} - \frac{1}{3(s^2+4)}$

$= e^{-2t} - \frac{1}{6}\sin 2t$

III $\frac{dy}{dt} - 4y = 8$

$Y'(t) - 4Y(s) = 8$

$Y'(t) - 4y(t) = 8$

$sY(s) - Y(0) - 4Y(s) = \frac{8}{s}$

$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$

$Y(s) = \frac{2s+8}{s(s-4)}$

$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$2s+8 = A(s-4) + Bs$

$2(0)+8 = A(0-4) \Rightarrow A = -2$

$2(4)+8 = B(4) \Rightarrow B = 4$

$\therefore L\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$

IV $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$

$Y''(t) - 2y'(t) + 5y(t) = e^{2t}$

$L[Y''(t)] = s^2Y(s) - sY(0) - Y'(0)$

$L[Y'(t)] = sY(s) - Y(s)$

$L(Y(t)) = Y(s)$

$s^2Y(s) - sY(s) - Y'(s) - 2sY(s) + 2Y(s) + 5Y(s) = \frac{1}{s-2}$

$(s^2-2s+5)Y(s) + (2-s)y_0 - y'(0) = \frac{1}{s-2}$

$(s^2-2s+5)Y(s) = \frac{1}{s-2} - (2-s)*2 + 1$

$= \frac{1}{s-2} + 2s - 4 + 1 = \frac{1}{s-2} + 2s - 3$

$= \frac{1+(2s-3)(s-2)}{(s-2)} = \frac{1+2s^2-4s-3s+6}{s-2}$

$Y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$

$2s^2-7s+7 = \frac{A}{s-2} + \frac{B}{s^2-2s+5}$

$2s^2-7s+7 = A(s^2-2s+5) + B(s-2)$

at $s = 2$

$2(2)^2-7(2)+7 = A(2^2-2(2)+5)$

$A = 2$

$-2A+B = -7$

$B = -7+4 = -3$

$= \frac{2}{s-2} - \frac{3}{s-2s+5}$

$= 2e^{2t} - \frac{1}{3}t\sin 2t$

v $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0) + 8Y(s) = \frac{1}{s-3}$

$[s^2-6s+8)Y(s) + (6-s)y(0) - y'(0) = \frac{1}{s-3}$

$[s^2-6s+8)Y(s) = \frac{1}{s-3} + 2 = \frac{1+2s-6}{s-3} = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

$2s-5 = A(s^2-6s+8) + B(s^2-7s+12) + C(s^2-5s+6)$

at $s = 3$

$2(3)-5 = A(3-2)(3-4)$

$A = -1$

at $s = 4$

$2(4) - 5 = 3 = C(4-3)(4-2)$

$C = 3/2$

$-6A - 7B - 5C = 7$

$-6(-1) - 7(B) - 5\left(\frac{3}{2}\right) = 2$

$-7B = \frac{2+15}{2} - 6 = \frac{4+15-12}{2} = \frac{7}{2}$

$B = -\frac{1}{2}$

$L^{-1}\left[\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}\right]$

$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$