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* Assignment V

Course:- ENG381 <Engineering Mathematics III>

1) $dy/dt + 3y = e^{-2t}$ given that at $t=0$, $y=2$

$$L\{dy/dt\} = SY(s) - Y(0)$$

$$L\{3y\} = 3\{Y(s)\}$$

$$L\{e^{-2t}\} = \frac{1}{S+2}$$

$\therefore SY(s) - Y(0) + 3Y(s) = \frac{1}{S+2}$

$\Rightarrow$ at $t=0$, $y=2$

$$SY(s) - 2 + 3Y(s) = \frac{1}{S+2}$$

$$SY(s) + 3Y(s) = \frac{1}{S+2} + 2$$

$$Y(s)\{S+3\} = \frac{1+2\{S+2\}}{S+2}$$

$$Y(s)\{S+3\} = \frac{1+2S+4}{S+2}$$

$$Y(s)\{S+3\} = \frac{2S+5}{S+2}$$

$$Y(s) = \frac{2S+5}{\{S+2\}\{S+3\}}$$

Using partial fraction.

$$\frac{2S+5}{\{S+2\}\{S+3\}} = \frac{A}{S+2} + \frac{B}{S+3}$$

$$2S+5 = A\{S+3\} + B\{S+2\}$$

Assuming $S+3=0$

$$S=-3.$$

$$2\{-3\} + 5 = A\{-3+3\} + B\{-3+2\}$$

$$-6+5 = B\{-1\}$$

$$-1 = -B$$

$$B=1$$

Assuming $S+2=0$

$$S=-2.$$

$$2\{-2\} + 5 = A\{-2+3\} + B\{-2+2\}$$

$-4+5 = A\{1\} + B(0)$

$1 = A\{1\}$

$A = 1$

$\therefore \quad Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{1}{s+2}\right\} + L^{-1}\left\{\frac{1}{s+3}\right\}$

$y = e^{-2t} + e^{-3t}$

2 $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t=0$, $y=1$

$\mathcal{L}\{3\frac{dy}{dt}\} = 3\{sY(s) - Y(0)\}$

$\mathcal{L}\{-6y\} = -6\{Y(s)\}$

$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+2^2}$

$3sY(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+4}$

at $t=0$, $y=1$

$3sY(s) - (3\times 1) - 6Y(s) = \frac{2}{s^2+4}$

$3sY(s) - 3 - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)\{3s-6\} = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$

$Y(s)\{3s-6\} = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+4)}$

$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$

Using partial fraction.

$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{(3s-6)}$

[C] $\left.\frac{3s^2+14}{s^2+4}\right|_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$

$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$

$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

Comparing Co-efficient.

$3 = 3B+C$

$3 = 3B + 13/4$

$3A - 6B = 0$

$3A = 6B$ $\therefore A = -1/6$

$A = 6 \times -1/12$

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$3B = -1/4$

$B = -1/12$

$$Y(s) = \frac{-1/6 - \{-\frac{1}{12}\}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$= \frac{-1/6}{s^2+4} - \frac{\frac{1}{12}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$= -\frac{1}{6} \cdot \frac{1}{s^2+2^2} - \frac{1}{12} \cdot \frac{s}{s^2+2^2} + \frac{13}{4} \cdot \frac{1}{3(s-2)}$$

$$= -\frac{1}{6} \cdot \frac{1}{2}\left\{\frac{2}{s^2+2^2}\right\} - \frac{1}{12}\left\{\frac{s}{s^2+2^2}\right\} + \frac{13}{12}\left\{\frac{1}{s-2}\right\}$$

$$y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{1}{12}\{-\sin 2t - \cos 2t + 13e^{2t}\}$$

Re-arranging

$$y(t) = \frac{1}{12}\{13e^{2t} - \cos 2t - \sin 2t\}$$

3) $dy/dt - 4y = 8$ at $t=0$, $y=2$

$y' - 4y = 8$

$S\,y(s) - y(0) - 4y(s) = \frac{8}{S}$

Applying the condition:

$y(s)[S-4] = \frac{8}{S} + \frac{2}{1} = \frac{8+2S}{S}$

$y(s) = \frac{8+2S}{S\{S-4\}} = \frac{A}{S} + \frac{B}{S-4}$

$\left.\frac{8+2S}{S-4}\right|_{S=0} = \frac{8}{-4} = -2$

$\left.\frac{8+2S}{S}\right|_{S=4} = \frac{8+2(4)}{4} = 4$

$y(s) = \frac{-2}{S} + \frac{4}{S-4}$

$y(t) = -2 + 4e^{4t}$

4) $d^2y/dx^2 - 2\,dy/dx + 5y = e^{2t}$ at $t=0$ $y=2$, $y^{(1)}=1$

$y'' - 2y' + 5y = e^{2t}$

$\{S^2y(s) - S\,y(0) - y'(0)\} - 2\{S\,y(s) - y(0)\} + 5\,y(s) = \frac{1}{S-2}$

$S^2y(s) - 2S - 1 - 2S\,y(s) + 4 + 5\,y(s) = \frac{1}{S-2}$

$y(s)[S^2 - 2S + 5] = \frac{1}{S-2} + \frac{2S}{1} - \frac{3}{1} = \frac{1+2S\{S-2\} - 3\{S-2\}}{S-2}$

$y(s)[S^2 - 2S + 5] = \frac{1+2S^2-4S-3S+6}{S-2} = \frac{2S^2-7S+7}{S-2}$

$y(s) = \frac{2S^2-7S+7}{(S-2)(S^2-2S+5)}$

Using partial fraction:

$\frac{2S^2-7S+7}{(S-2)(S^2-2S+5)} = \frac{A}{S-2} + \frac{BS+C}{S^2-2S+5}$

$[A] = \left.\frac{2S^2-7S+7}{S^2-2S+5}\right|_{S=2} = \frac{2(2)^2-7(2)+7}{2^2-2(2)+5} = \frac{1}{5}$

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$2s^2 - 7s + 7 = A\{s^2 - 2s + 5\} + \{Bs + C\}\{s - 2\}$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2 = A + B$

$2 = 1/5 + B$

$B = 9/5$

$7 = 5A - 2C$

$7 = 5\{1/5\} - 2C$

$7 - 1 = -2C$

$C = -3$

$$y(s) = \frac{1/5}{s-2} + \frac{\frac{9}{5}s - 3}{s^2 - 2s + 5} = \left\{\frac{1}{5} \cdot \frac{1}{s-2}\right\} + \frac{\frac{9}{5}s}{\{s+1\}^2 + 4} - \frac{3}{(s+1)^2}$$

$$y(s) = \left\{\frac{1}{5} \cdot \frac{1}{s-2}\right\} + \frac{9}{5}\left\{\frac{s-1+1}{\{s+1\}^2 + 4}\right\} - \frac{3}{(s+1)^2 + 2^2}$$

$$y(s) = \left\{\frac{1}{5} \cdot \frac{1}{s-2}\right\} + \frac{9}{5} \cdot \frac{s+1}{(s+1)^2 + 2^2} - \frac{4}{(s+1)^2 + 4} - \frac{3}{(s+1)^2 + 2^2}$$

$$y(s) = \left\{\frac{1}{5} \cdot \frac{1}{s-2}\right\} + \frac{9}{5} \cdot \frac{s+1}{(s+1)^2 + 2^2} - \frac{4}{(s+1)^2 + 2^2} \cdot \frac{2}{2}$$

$$y(s) = \left\{\frac{1}{5} \cdot \frac{1}{(s-2)}\right\} + \left\{\frac{9}{5} \cdot \frac{s+1}{(s+1)^2 + 2^2}\right\} - 2 \cdot \frac{2}{(s+1)^2 + 2^2}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 2t - 2e^{-t}\sin 2t$$

$$y(t) = \frac{1}{5}\{e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t\}$$

$$= 1/5\{e^{2t} + e^{-t}\{9\cos 2t - 10\sin 2t\}\}$$

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5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$, $y'=2$

$y'' - 6y' + 8y = e^{3t}$

$s^2 y(s) - s y(0) - y'(0) - 6\{s y(s) - y(0)\} + 8 y(s) = \frac{1}{s-3}$

At $t=0$, $y=0$, $y'=2$

$s^2 y(s) - 2 - 6s y(s) + 8 y(s) = \frac{1}{s-3}$

$y(s)\{s^2 - 6s + 8\} = \frac{1}{s-3} + \frac{2}{1}$

$y(s)\{s^2 - 6s + 8\} = \frac{(1 + 2(s-3))}{s-3}$

$y(s)\{s^2 - 6s + 8\} = \frac{1 + 2s - 6}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)}$

Using Partial fraction.

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} + \frac{C}{(s-4)}$

$[A] = \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$

$[B] = \left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = \frac{-1}{2}$

$[C] = \left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$

$y(s) = \left\{\frac{-1}{s-3}\right\} - \left\{\frac{1}{2}\cdot\frac{1}{s-2}\right\} + \left\{\frac{3}{2}\cdot\frac{1}{s-4}\right\}$

$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$.