Assignment 5

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1) $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0$, $y=2$

$L\{dy/dt\} = sY(s) - y(0)$

$L\{3y\} = 3y(s)$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sY(s) - y(0) - 3y(s) = \frac{1}{s+2}$

$sY(s) + 3y(s) - 2 = \frac{1}{s+2}$

$(s+3)y(s) = \frac{1}{s+2} + 2$

$(s+3)y(s) = \frac{1+2(s+2)}{s+2}$

$(s+3)y(s) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As + 3A + Bs + 2B$

$A + B = 2$ ... ① ×3

$3A + 2B = 5$ ... ② ×1

$\Rightarrow 3A + 3B = 6$ ... ③

$3A + 2B = 5$ ... ④

eqn ③ − ④

$B = 1$

sub $b=1$ in eqn 1

$A + 1 = 2$

$A = 1$

$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = 2\sin 2t$ at $t=0$, $y=1$

$L\{3\frac{dy}{dt}\} = 3(sy(s) - y(0))$

$L\{-6y\} = -6y(s)$

$L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$

$3sy(s) - 6y(s) - 3y(0) = \frac{2}{s^2+2^2}$

at $y=1$

$(3s-6)y_s - 3 = \frac{2}{s^2+2^2}$

$(3s-6)y_s = \frac{2}{s^2+2^2} + 3$

$(3s-6)y(s) = \frac{2+3(s^2+2^2)^2}{(s^2+2)^2}$

$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

$2+3s^2+12s+12 = A3s^2 - 12A + 3Bs - 6B - Cs + 4Cs + 4C$

$3A + C = 3$ .. ①

$3B + 4C = 12$ .. ②

$-12A - 6B + 4C = 14$

from eqn (1)

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C - 6B + 4C = 14 \times -6$ .. ③

$-6B + 8C = 28$ .. ×3 .. ④

$-18B - 24C = -72$ .. ⑤

$-18B + 24C = 84$ .. ⑥

④+⑤

$48C = -156$

$C = 13/4$

from eqn ②

$3B = 12 - 13$

$3B = -1$

$B = -1/3$

from eqn ①

$3A = 3 + C$

$3A = 3 - \frac{13}{4}$

$A = -1/12$

$\frac{(2s+3)(s+2)^2}{(s+2)^2(3s-6)} = \frac{-1/12}{s+2} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(2s-6)}$

$\therefore y(t) = \frac{-1}{12}e^{-2t} - \frac{1}{3}te^{-2t} + \frac{13}{12}e^{2t}$

$\therefore y(t) = \frac{1}{12}(e^{-2t} + 4te^{-2t} - 13e^{2t})$

3 $\frac{dy}{dt} - 4y = 8$ at $t=0, y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{-4y\} = -4y(s)$

$L\{8\} = 8/s$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 4y(s) - y(0) = 8/s$

$y(s)(s-4) - 2 = \frac{8}{s}$

$(s-4)y(s) = \frac{8}{s} + 2$

$(s-4)y(s) = \frac{8+2s}{s}$

$ys = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$8+2s = A(s-4) + Bs$

$8+2s = As - 4A + Bs$

$A+B = 2$

$-4A = 8$

$A = -2$

$B = 2+2 = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$y(t) = -2 + 4e^{4t}$

4) $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$ at $y=3, t=1$

$L\{d^2y/dt^2\} = s^2y(s) + sy(0) + y'(0)$

$L\{2dy/dt\} = -2sy(s) + 2y(0)$

$L\{5y\} = 5y(s)$

$L\{e^{2t}y\} = 1/s+2$

$s^2y(s) - sy(0) - y'(0) - 2sy(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$(s^2 - 2s + 5)y(s) = \frac{1}{s-2} + 2s - 3$

$y(s)(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-2)}$

$y(s) = \frac{1 + 2s^2 - s + 6}{(s-2)(s^2 - 2s + 5)}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs+C}{(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$A + B = 2 \quad \cdots ①$

$-2A - 2B + C = -7 \quad \cdots ②$

$5A - 2C = 7 \quad \cdots ③$

from ①

$B = 2 - A$

from ②

From ②

$-2A - 2(2-A) + C = -7$

$-2A - 4 + 2A + C = -7$

$C = -3$

From ⑤

$5A - 2(-3) = 7$

$5A = 7 - 6$

$A = 1/5$

$A + B = 2$

$\frac{1}{5} + B = 2$

$B = 2 - 1/5$

$B = 9/5$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{1/5}{s-2} + \frac{9/5(s-3)}{s^2-2s+5}$

$= \frac{1}{5} \cdot \frac{1}{s+2} + \frac{9}{5} \frac{s}{s^2-2s+5} - \frac{3}{(s^2-2s+5)}$

$= \frac{1}{5} \frac{1}{(s+2)} + \frac{9}{5}\left(\frac{s-1+1}{s^2-2s+5}\right) - \frac{3 \times \frac{2}{2}}{s^2-2s+5}$

$= \frac{1}{5} \frac{1}{s+2} + \frac{9}{5} \frac{(s-1+1)}{(s-1)^2+4} - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$

$= \frac{1}{5} \frac{1}{s-2} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1 \times \frac{2}{2}}{(s-1)^2+2^2}\right] - \frac{3}{2}\left[\frac{2}{(s-1)^2+2^2}\right]$

$= L^{-1}\left\{\frac{1}{5} \frac{1}{s-2} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right] - \frac{3}{2}\left[\left(\frac{2}{(s-1)^2+2^2}\right)\right]\right\}$

$y(t) = \frac{1}{5} e^{2t} + \frac{9}{5}\left[e^t \cos 2t + \frac{1}{2}(e^t \sin 2t)\right] - \frac{3}{2}(e^t \sin 2t)$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0, y=0, y'=2$

$L\{d^2y/dt^2\} = s^2 y(s) - sy(0) - y'(0)$

$L\{-6dy/dt\} = -6sy(s) + 6y(0)$

$L\{8y\} = 8y(s)$

$L\{e^{3t}y\} = 1/s-3$

$s^2y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$

$y(s)(s^2-6s+8) = \frac{1}{s-3} + 2$

$(s^2-6s+8)y(s) = \frac{1+2(s-3)}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{s-3} + \frac{Bs+C}{s^2-6s+8}$

$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$

$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$

$A + B = 0$

$-6A - 3B + C = 0$

$8A - 3C = -3$

$B = -A$ from eqn ①

$-6A + 3A + C = 2$

$-3A + C = 2 \quad \cdots ④ \times -3$

$8A - 3C = -5 \quad \cdots ⑤ \times 1$

$9A - 3C = -6$

$8A - 3C = -5$

$A = -1$

$B = -1$

from $C = 2 - 3 = -1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{s-1}{s^2-6s+8}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$s-1 = A(s-4) + B(s-2)$$

$$s-1 = As - 4A + Bs - 2B$$

$$A + B = 1 \quad \text{---} (1) \times -4$$

$$-4A - 2B = -1 \quad \text{---} (2) \times 1$$

$$-4A - 4B = -4$$

$$-4A - 2B = -1$$

$$-2B = -3$$

$$B = 3/2$$

$$A = -1/2$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \left[\frac{-1/2}{s-2} + \frac{3/2}{s-4}\right]$$

$$\therefore y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$

$$y(t) = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$$