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15/ENG07/031
PETROLEUM ENGINEERING
ENG 381 ASSIGNMENT IV

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

General Equation

$$W^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + n(n-1)(n-2) u^{n-3} v''' + \cdots$$

$(1-x^2)y'' - 2xy' + 2y = 0$

For ; $(1-x^2)y''$

let $u = y''$, $u' = y'''$, $u'' = y^{iv}$; $u^n = y^{n+2}$

$V = V^0 = 1-x^2$, $V' = -2x$, $V'' = -2$, $V''' = 0$

$W_1^n = y^{n+2}(1-x^2) + n y^{n+1}(-2x) + \left(\frac{n(n-1)}{2}\right) y^n (-2) + 0$

$W_1^n = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n$

For ; $-2xy'$

let $u = y'$, $u' = y''$, $u'' = y'''$; $u^n = y^{n+1}$

$V = -2x$, $V' = -2$, $V'' = 0$

$W_2^n = y^{n+1}(-2x) + ny^n(-2) + 0$

$= -2xy^{n+1} - 2ny^n$

For ; $2y$

let $u = y$; $u^n = y^n$

$V = 2$; $V' = 0$

$W_3^n = 2y^n$

$W_n = W_1^n + W_2^n + W_3^n$

$W_n = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

at $x = 0$

$(1-x^2)y^{n+2} - (2xn + 2x)y^{n-1} - (n^2 - n + 2n - 2)y^n = 0$

$(1-x^2)y^{n+2} - (2xn + 2x)y^{n-1} - (n^2 + n - 2)y^n = 0$

$y^{n+2} - 0 - (n^2 + n - 2)y^n = 0$

$y^{n+2} = (n^2 + n - 2)(y^n)_0$

When $n = 0$

$(y^2)_0 = y'' = -2(y^0)_0$

When $n = 1$

$(y^3)_0 = y''' = 0(y')_0 = 0$

When $n = 2$

$(y^4)_0 = y^{iv} = 4(y^2)_0 = 4 \times -2(y^0)_0 = -8(y^0)_0$

When $n=3$

$(y^5)_0 = y^{v} = 10(y^3)_0 = 10 \times 0 = 0$

When $n=4$

$(y^6)_0 = y^{vi} = 18(y^4)_0 = 18 \times -8(y^0)_0 = -144(y^0)_0$

When $n=5$

$(y^7)_0 = y^{vii} = 28(y^5)_0 = 28 \times 0 = 0$

When $n=6$

$(y^8)_0 = y^{viii} = 40(y^6)_0 = 40 \times -144(y^0)_0 = -5760(y^0)_0$

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \cdots$

$y = (y)_0 + x(y')_0 + \frac{x^2}{2!}(-2(y^0)_0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-8(y^0)_0) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(-144(y^0)_0) + \frac{x^7}{7!}(0) + \frac{x^8}{8!}(-5760(y^0)_0)$

$y = (y)_0 + x(y')_0 + (-x^2)(y^0)_0 + 0 - \frac{x^4}{3}(y^0)_0 + 0 + (-\frac{x^6}{5})(y^0)_0 + 0 + (\frac{-x^8}{7}(y^0)_0) + \cdots$

$\therefore y = (y^0)_0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7}\right] + (y')_0 x$

2i $L(3e^{-4t} - 5e^{4t}) =$

$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$

$= \frac{3s - 12 - 5s - 20}{s^2 - 16} = \frac{-2s - 32}{s^2 - 16}$

ii $L(\sin 4t + \cos 4t)$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$

iii $L(t^3 + 2t^2 - t + 4)$

$= \frac{3!}{s^4} + \frac{2(2!)}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6 + 4s - s^2 + 4s^3}{s^4}$

iv $L(e^{-2t}\cos 5t) = \frac{s+2}{(s+2)^2 + 5^2} = \frac{s+2}{s^2 + 4s + 29}$

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v $L(t\sin 3t) = (-1)^n \cdot \frac{d^n}{dx^n}[f(t)] =$

$$-1\frac{d}{ds}\left[\frac{3}{s^2+3^2}\right] = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$$

Using quotient rule

$u = 3 \qquad v = s^2+9$

$\frac{du}{ds} = 0 \qquad \frac{dv}{ds} = 2s$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{0 - 6s}{(s^2+9)^2}$$

$$L(t\sin 3t) = \frac{+6s}{(s^2+9)^2}$$

vi $$L\left[\frac{e^{-t}-e^{-2t}}{t}\right] = \frac{\left[\frac{1}{s+1} - \frac{1}{s+2}\right]}{\frac{1}{s^{1+1}}} = \left[\frac{1}{s+1} - \frac{1}{s+2}\right]s^2$$

$$= \frac{s^2}{(s+1)(s+2)}$$

vii $L(e^{4t}\cos 2t)$

$$L(\cos 2t) = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

Let $s = s-4$

$$L(e^{4t}\cos 2t) = \frac{s-4}{(s-4)^2+4} = \frac{s-4}{s^2-8s+20}$$

viii $$L(t\sin 2t) = -1\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$$

$u = 2 \qquad v = s^2+4$

$\frac{du}{ds} = 0 \qquad \frac{dv}{ds} = 2s$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2} = \frac{-4s}{(s^2+4)^2}$$

$$\therefore L(t\sin 2t) = \frac{4s}{(s^2+4)^2}$$

ix $t^3 + 4t^2 + 5 = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+1}} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x $L[e^{3t}(t^2+4)] = L(t^2e^{3t} + 4e^{3t})$

$= \frac{2!}{(s-3)^{2+1}} + \frac{4}{s-3} = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

$= \frac{2(s-3) + 4(s-3)^2}{(s-3)^4} = \frac{2s - 6 + 4(s^2 + 9 - 6s)}{(s-3)^4}$

$= \frac{2s - 6 + 4s^2 + 36 - 24s}{(s-3)^4} = \frac{4s^2 - 22s + 30}{(s-3)^4}$

xi $t^2 \cos t = (-1)^2 \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right] = \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{s}{s^2+1}\right)\right]$

$u = s \; ; \; \frac{du}{ds} = 1$

$v = s^2 + 1 \; ; \; \frac{dv}{ds} = 2s$

$\therefore \frac{d}{ds}\left(\frac{s}{s^2+1}\right) = \frac{(s^2+1)(1) - 2s(s)}{(s^2+1)^2} = \frac{s^2 + 1 - 2s^2}{(s^2+1)^2} = \frac{1 - s^2}{(s^2+1)^2}$

$\frac{d}{ds}\left[\frac{d}{ds}\left(\frac{s}{s^2+1}\right)\right] = \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right]$

$u = 1 - s^2 \; ; \; \frac{du}{ds} = -2s$

$v = s^4 + 2s^2 + 1 \; ; \; \frac{dv}{ds} = 4s^3 + 4s$

$\therefore \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right] = \frac{[(s^4 + 2s^2 + 1)(-2s)] - [(1-s^2)(4s^3+4s)]}{(s^2+1)^2}$

$= \frac{-2s^5 - 4s^3 - 2s - (4s^3 + 4s - 4s^5 - 4s^3)}{(s^2+1)^2}$

$= \frac{2s^5 - 4s^3 - 6s}{(s^2+1)^2}$

xii $\frac{\sinh 2t}{t} = \frac{1}{2} \ln(s^2 - 4) - \ln s$

3. i $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

At $s = 3$;

$3-5 = A(3-4) + B(3-3)$

$-2 = -A$

$\therefore A = 2$

At $s = 4$;

$4-5 = A(4-4) + B(4-3)$

$-1 = B$; $B = -1$

$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$

$\therefore L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right] = 2e^{3t} - e^{4t}$

ii $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

At $s = 2$; $2(2) - 6 = A(2-4) + B(2-2)$

$4-6 = -2A$

$A = \frac{-2}{-2} = 1$

At $s = 4$; $2(4) - 6 = A(4-4) + B(4-2)$

$8-6 = 2B$

$B = 2/2 = 1$

$\therefore \frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$

$L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right] = e^{2t} + e^{4t}$

iii $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

At $s = 0$; $5(0) - 8 = A(0-4) + B(0)$

$$-8 = -4A$$
$$A = 2$$
At $s = 4$; $5(4) - 8 = B(4)$
$$B = 3$$
$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$
$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right] = 2 + 3e^{4t}$$

iv $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$
$$s^2 - 3s - 4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$$
At $s = 3$; $3^2 - 3(3) - 4 = A(3-1)^2 + B(3-1)(3-3) + C(3-3)$
$$-4 = 4A$$
$$A = -1$$
At $s = 1$; $1^2 - 3(1) - 4 = A(1-1)^2 + B(1-1)(1-3) + C(1-3)$
$$-6 = -2C$$
$$C = 3$$
$$s^2 - 3s - 4 = [s^2 - 2s - 1]A + (s^2 - 4s + 3)B + (s-3)C$$
$$-2A - 4B + C = -3$$
$$-2(-1) - 4B + 3 = -3$$
$$2 - 4B + 3 = -3$$
$$8 = 4B$$
$$B = 2$$
$$\therefore \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$
$$L^{-1}\left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right] = -e^{3t} + 2e^{t} + 3te^{t}$$

v $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+4^2}$$
$$= \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{16}$$

$= A(s+2)(16) + B(16) + C(s+2)^2 = s-5$

At $s=\cancel{2}-2$; $B(16) = -2-5 = -7$

$B = {}^{-7}/16$

from the eqt: $A(16s+32) + 16B + (s^2+4s+4)C = s-5$

$16A + 4C = 1$

$-\quad 32A + 4C + 16B = 5$

$-16A - 16B = +6$

$-16A - 16(-7/16) = +6$

$-16A = +6 - 7 = -1$

$A = 1/16$

$16(1/16) + 4C = 1$

$4C = 1 - 1$

$\therefore C = 0$

$\therefore \cancel{\dfrac{s^2-3s-4}{(s-3)(s-1)^2} = \dfrac{1}{16(s-3)} - \dfrac{7}{16(s-1)} + \dfrac{0}{(s-1)^2}}$

$\therefore \dfrac{s-5}{s^2+4s+20} = \dfrac{1}{16(s+2)} - \dfrac{7}{16(s+2)^2} + \dfrac{0}{16}$

$L^{-1}\left[\dfrac{1}{16(s+2)} - \dfrac{7}{16(s+2)^2}\right] = 1/16\, e^{-2t} - 7/16\, t e^{2t}$