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15/ENG07/031
PETROLEUM ENGINEERING
ENG381 ASSIGNMENT V

1. $\frac{dy}{dt} + 3y = e^{-3t}$, Given that at $t = 0$, $y = 2$

$y'(t) + 3y(t) = e^{-2t}$

$L[y'(t)] = sy(s) - y(0)$

$L[y(t)] = Y(s)$

$L^{-1}(e^{-2t}) = \frac{1}{s+2}$

$\therefore sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$(s-3)Y(s) - 2 = \frac{1}{s+2}$

$(s-3)Y(s) = \frac{1}{s+2} + 2$

$(s-3)Y(s) = \frac{1 + 2s + 4}{s+2} = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2s+5 = A(s-3) + B(s+2)$

Where $s = 3$

$2(3) + 5 = A(3-3) + B(3+2)$

$11 = 5B$

$B = 11/5$

Where $s = -2$

$2(-2) + 5 = A(-2-3)$

$1 = -5A$

$A = -1/5$

$\therefore Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)}$

$= \frac{-1}{5}e^{-2t} + \frac{11}{5}e^{3t}$

ii. $3\frac{dy}{dt} - 6y = \sin 2t$, Given that at $t = 0$, $y = 1$

$3y'(t) - 6y(t) = \sin 2t$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$

$L(\sin 2t) = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3sY(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$(3s-6)Y(s) - 3 = \frac{2}{s^2+4}$

$(3s-6)Y(s) = \frac{2}{s^2+4} + 3 = \frac{2 + 3s^2 + 12}{s^2+4}$

$(3s-6)Y(s) = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$

$3s^2+14 = A(s^2+4) + B(3s-6)$

$4A - 6B = 14 \quad \cdots ①$

$A = 3$

$4(3) - 6B = 14$

$12 - 14 = 6B \quad ; \quad -2 = 6B$

$B = -1/3$

$\therefore Y(s) = \frac{3}{3s-6} - \frac{1}{3(s^2+4)} = \frac{1}{s-2} - \frac{1}{3(s^2+4)}$

$= e^{-2t} - \frac{1}{3}(\sin 2t)$

iii) $\frac{dy}{dt} - 4y = 8$; Given that $t = 0, y = 2$

$y'(t) - 4y(t) = 8$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$

$L(8) = \frac{8}{s}$

$\therefore sY(s) + y(0) - 4Y(s) = \frac{8}{s}$

$(s-4)Y(s) - 2 = \frac{8}{s}$

$(s-4)Y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$

$Y(s) = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$2s+8 = A(s-4) + B(s)$

When $s = 0$; $2(0)+8 = A(0-4) + B(0)$

$8 = -4A$

$A = -2$

When $s = 4$; $2(4)+8 = A(4-4) + B(4)$

$16 = 4B$

$B = 4$

$\therefore Y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$

iv $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$; Given that $t=0; y=2, y'=1$

$y''(t) - 2(y')_t + 5y(t) = e^{2t}$

$L[(y'')_t] = s^2Y(s) - sY(0) + y'(0)$

$L[y'(t)] = sY(s) - Y(0)$

$L(Y(t)) = Y(s)$; $L(e^{2t}) = \frac{1}{s-2}$

$s^2Y(s) - sY(0) - y'(0) - 2sY(s) + 2Y(0) + 5Y(s) = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(s) + (2-s)Y(0) - y'(0) = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(s) + (2-s)2 - 1 = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(s) + 4 - 2s - 1 = \frac{1}{s-2}$

$(s^2 - 2s + 5)Y(s) = \frac{1}{s-2} - 3 + 2s$

$(s^2 - 2s + 5)Y(s) = \frac{1 - 3(s-2) + 2s(s-2)}{s-2}$

$(s^2 - 2s + 5)Y(s) = \frac{2s^2 - 7s + 7}{s-2}$

$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{B + C}{s^2 - 2s + 5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$

At $s = 2$; $2(2)^2 - 7(2) + 7 = A(2^2 - 2(2) + 5)$

$1 = 5A$

$A = 1/5$

$5A - 2B = 7$

$5(1/5) - 2B = 7$

$-6 = 2B$; $B = -3$

$\therefore Y(s) = \frac{1}{5(s-2)} - \frac{3}{s^2 - 2s + 5}$

V $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$; Given that $t=0, y=0, y'=2$

$y''(t) - 6y'(t) + 8y(t) = e^{3t}$

$L[y''(t)] = s^2Y(s) - sY(0) - y'(0)$

$L[y'(t)] = sY(s) - Y(0)$

$L[y(t)] = Y(s)$

$L(e^{3t}) = \frac{1}{s-3}$

$s^2Y(s) - sY(0) - y'(0) - 6sY(s) + 6Y(0) + 8Y(s) = \frac{1}{s-3}$

$(s^2 - 6s + 8)Y(s) + (6-s)Y_0 + (\cancel{8-6s})\cancel{Y(s)}\ (-y'(0)) = \frac{1}{s-3}$

$(s^2 - 6s + 8)Y(s) + (6-s)0 - 2 = \frac{1}{s-3}$

$(s^2 - 6s + 8)Y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3} = \frac{2s-5}{s-3}$

$Y(s) = \frac{2s-5}{s-3(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)}$

$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

Where $s=3$

$2(3)-5 = A(3-2)(3-4)$

$1 = -A$; $A = -1$

Where $s=2$

$2(2)-5 = B(2-3)(2-4)$

$-1 = 2B$; $B = -1/2$

Where $s=4$

$2(4)-5 = C(4-3)(4-2)$

$3 = 2C$; $C = 3/2$

$\therefore Y(s) = \frac{-1}{s-3} + \frac{-1}{2(s-2)} + \frac{3}{2(s-4)}$

$L^{-1}(Y(s)) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$