Assignment 5

Oyebanji: Olalekan Joshua
15/ENG04/049
Elect/Elect ENG.

1) $\frac{dy}{dt} + 3y = e^{-2t}$ given at $t=0$ $y=2$

$y(5) = 2$

$y^{(1)} + 3y^{(0)} = e^{-2t}$

$5y(5) - y(5) + 3y(5) = \frac{1}{5+2}$

$5y(5) - 2 + 3y(5) = \frac{1}{5+2}$

$y(5)[5+3) = \frac{1}{5+2} + \frac{2}{1} = \frac{1+2(5+2)}{5+2} = \frac{1+25+4}{5+2}$

$y(5) = \frac{1+25+4}{(5+2)(5+3)} = \frac{A}{5+2} + \frac{B}{5+3}$

A: $\left.\frac{1+25+4}{5+3}\right|_{5=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

B: $\left.\frac{1+25+4}{5+2}\right|_{5=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

$y(5) = \frac{1}{5+2} + \frac{1}{5+3}$

$y(t) = e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$

$y(5) = 1$

$3y^{(1)} - 6y = \sin 2t$

$3(5y(5) - y(5)) - 6y(5) = \frac{2}{5+4}$

$3Sy(5) - 3y(0) - 6y(5) = \frac{2}{5+4}$

$y(5)(35-6] = \frac{2}{5^2+4} + \frac{3}{1} = \frac{2+3(5^2+4)}{5^2+4}$

$y(5)[35-6] = \frac{2+35^2+12}{(5^2+4)} = \frac{35^2+14}{(5^2+4)}$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{(3s-6)}$$

C: $\left.\frac{3s^2+14}{s^2+4}\right|_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$

$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$

$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

Comparing coefficient.

$3 = 3BAC$

$3 = 3B + \frac{13}{4}$ (where $C = 13/4$)

$3A - 6B = 0$

$3A = 6B$

$3A = 6 \times \frac{-1}{12}$ $\quad A = -1/6$

$$y(s) = \frac{-1/6 - 1/12 s}{s^2+4} + \frac{13/4}{3s-6}$$

$$\frac{-1/6}{s^2+4} - \frac{1/12 s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= -\frac{1}{6}\cdot\frac{1}{s^2+2^2} - \frac{1}{12}\cdot\frac{s}{s^2+2^2} + \frac{13}{4}\cdot\frac{1}{3(s-2)}$$

$$= -\frac{1}{6}\cdot\frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{12}\left[\frac{1}{s-2}\right]$$

$$y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$$

$$y(t) = \frac{1}{12}\left[13e^{2t} - \cos 2t - \sin 2t\right]$$

3) $\frac{dy}{dt} - 4y = 8$ given that $t=0$ $y=2$ $y(0)=2$

$y(t) - 4y = 8$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y(s)[s-4] = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

A: $\left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$

B: $\left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

4) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2t}$ at $t=0$ $y=2$ $y'(t)=1$

$y(s)=2$ $y(0)=1$

$y'' - 2y' + 5y = e^{2t}$

$(s^2y(s) - sy(s) - y'(0)) - 2(sy(s) - y(s)) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$y(s)[s^2 - 2s + 5) = \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1} = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$

$y(s)[s^2 - 2s + 5) = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+6}{s^2-2s+5}$

A: $\left.\frac{2s^2 - 7s + 7}{s^2 - 2s - 5}\right|_{s=-2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2 = A + B$

$2 = \frac{1}{5} + B \Rightarrow B = \frac{9}{5}$

$7 = 5A - 2C$

$7 = 5\left(\frac{1}{5}\right) - 2C$

$7 - 1 = -2C$

$C = -3$

$$Y(s) = \frac{\frac{1}{5}}{s-2} + \frac{\frac{9}{5}s - 3}{s^2 - 2s + 5} = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{\frac{9}{5}s}{(s+1)^2+4} - \frac{3}{(s+1)^2}$$

$$Y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$Y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{s+1}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$Y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{s+1}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}\cdot\frac{3}{2}$$

$$Y(s) = \frac{1}{5}\cdot\frac{1}{(s-2)} + \frac{9}{5}\cdot\frac{s+1}{(s+1)^2+2^2} - 2\cdot\frac{2}{(s+1)^2+2^2}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 2t - 2e^{-t}\sin 2t$$

$$y(t) = \frac{1}{5}\left[e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t\right]$$

$$= \frac{1}{5}\left(e^{2t} + e^{-t}\left(9\cos 2t - 10\sin 2t\right)\right]$$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$ $y=0$ $y^{(1)}=2$

$$y^{(2)} - 6y^{(1)} + 8y = e^{3t}$$

$$s^2y(s) - sy(0) - y'(0) - 6\left[sy(s) - y(0)\right] + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$y(s)\left[s^2 - 6s + 8\right] = \frac{1}{s-3} + 2 = \frac{1+2(s-3)}{s-3} = \frac{1+2s-6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} - \frac{C}{s-4}$$

A: $\left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$

B: $\left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = -\frac{1}{2}$

C: $\left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$

$$y(s) = \frac{-1}{s-3} - \frac{1}{2}\cdot\frac{1}{s-2} + \frac{3}{2}\cdot\frac{1}{s-4}$$

$$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$