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ASSIGNMENT 5

(1) $dy/dt + 3y = e^{-2t}$ given that at $t=0$, $y=2$

$L[dy/dt] = sY(s) - y(0)$

$L[3y] = 3[Y(s)]$

$L[e^{-2t}] = \frac{1}{s+2}$

$\therefore sY(s) - Y(s) + 3Y(s) = \frac{1}{s+2}$

$\Rightarrow$ at $t=0$, $y=2$

$sY(s) - 2 + 3Y(s) = \frac{1}{s+2}$

$sY(s) + 3Y(s) = \frac{1}{s+2} + 2$

$Y(s)[s+3] = \frac{1+2[s+2]}{s+2}$

$Y(s)[s+3] = \frac{1+2s+4}{s+2}$

$Y(s)[s+3] = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{[s+2][s+3]}$

Using partial fraction

$\frac{2s+5}{[s+2][s+3} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A[s+3] + B[s+2]$

Assuming $s+3 = 0$

$s = -3$

$2[-3] + 5 = A[-3+3] + B[-3+2]$

$-6 + 5 = B[-1]$

$-1 = -B$, $B = 1$

Assuming $s+2 = 0$

$s = -2$

$2[-2] + 5 = A[-2+3] + B[-2+2]$

$-4 + 5 = A[1] + B[0]$

$A = 1$

$$Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}[Y(s)] = L^{-1}\left[\frac{1}{s+2}\right] + L^{-1}\left[\frac{1}{s+3}\right]$$

$$y = e^{-2t} + e^{-3t}.$$

(2) $3\,dy/dt - 6y = \sin 2t$ given that at $t=0,\ y=1$

$$L[3\,dy/dt] = 3[sY(s) - Y(0)]$$

$$L[-6y] = -6[Y(s)]$$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$3sY(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2+4}$$

at $t=0,\ y=1$

$$3sY(s) - (3\times 1) - 6Y(s) = \frac{2}{s^2+4}$$

$$3sY(s) - 3 - 6Y(s) = \frac{2}{s^2+4}$$

$$Y(s)[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} = 2+3(s^2+4)/s^2+4$$

$$Y(s)[3s-6] = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$$

$$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{(3s-6)}$$

$$3s^2+14 \Rightarrow A+Bs(3s-6) + C(s^2+4)$$

at $s = 2$

$$8C = 26 \;;\quad C = 13/4$$

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$$

$$3 = 3B + C$$

$$3 = 3B + 13/4$$

$$3A - 6B = 0$$

$$3A = 6\times -1/12 \;\therefore\; A = -1/6$$

$$3B = -1/4$$

$$B = -1/12$$

$$Y(s) = \frac{-1/6 - [-1/12]s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= -\frac{1}{6}\cdot\frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{12}\left[\frac{1}{s-2}\right]$$

$y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$

$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$

$y(t) = \frac{1}{12}\left[13e^{2t} - \cos 2t - \sin 2t\right]$

(3) $dy/dt - 4y = 8$ at $t=0,\ y=2$

$y' - 4y = 8$

$sy(s) - y(0) - 4y(s) = 8/s$

$y(s)[s-4] = 8/s + \frac{2}{1} = 8+2s/s$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$

$\left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$

$y(s) = -2/s + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

(4) $d^2y/dx^2 - 2\,dy/dx + 5y = e^{2t}$ at $t=0$ $y=2,\ y'=1$

$y'' - 2y' + 5y = e^{2t}$

$[s^2y(s) - sy(0) - y'(0)] - 2[sy(s) - y(0)] + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = 1/s-2$

$y(s)[s^2 - 2s + 5] = 1/s-2 + \frac{2s}{1} - \frac{3}{1} = \frac{1+2s[s-2]-3[s-2]}{s-2}$

$y(s)[s^2-2s+5] = \frac{1+2s^2-4s-3s+6}{s-2} = \frac{2s^2-7s+7}{s-2}$

$y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$

$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$

$[A]\ \left.\frac{2s^2-7s+7}{s^2-2s+5}\right|_{s=2} = \frac{2(2)^2-7(2)+7}{2^2-2(2)+5} = \frac{1}{5}$

$2s^2 - 7s + 7 = A[s^2-2s+5] + [Bs+C][s-2]$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2 = A + B$

$2 = 1/5 + B$

$B = 9/5$

$7 = 5A - 2C$

$7 = 5(1/5) - 2C$

$C = -3$

$$Y(s) = \frac{1/5}{s-2} + \frac{9/5 s - 3}{s^2 - 2s + 5} = \left[\frac{1}{5} \cdot \frac{1}{s-2}\right] + \frac{9/5 s}{[s+1]^2 + 4} - \frac{3}{(s+1)^2}$$

$$Y(s) = \left[\frac{1}{5} \cdot \frac{1}{s-2}\right] + \frac{9}{5}\left[\frac{s-1+1}{[s+1]^2+4}\right] - \frac{3}{(s+1)^2 + 2^2}$$

$$Y(s) = \left[\frac{1}{5} \cdot \frac{1}{s-2}\right] + \left[\frac{9}{5} \cdot \frac{s+1}{(s+1)^2+2^2}\right] - 2 \cdot \frac{2}{(s+1)^2 + 2^2}$$

$$Y(t) = \frac{1}{5}e^{2t} + 9/5 e^{-t}\cos 2t - 2e^{-t}\sin 2t$$

$$Y(t) = \frac{1}{5}\left[e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t\right]$$

(5) $d^2y/dt^2 - 6\, dy/dt + 8y = e^{3t}$ at $t=0$, $y=0$, $y'=2$

$y'' - 6y' + 8y = e^{3t}$

$s^2 Y(s) - sy(0) - y'(0) - 6[sY(s) - y(0)] + 8Y(s) = \frac{1}{s-3}$

At $t=0$, $y=0$, $y'=2$

$s^2 Y(s) - 2 - 6sY(s) + 8Y(s) = \frac{1}{s-3}$

$Y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + \frac{2}{1}$

$$Y(s)[s^2 - 6s + 8] = \frac{1 + 2(s-3)}{s-3}$$

$$Y(s)[s^2 - 6s + 8] = \frac{1 + 2s - 6}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{B}{(s-2)} + \frac{C}{(s-4)}$$

$$[A]: \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3} = \frac{2(3)-5}{(3-3)(3-4)} = -\frac{1}{2} \times 2 = -1$$

$$[B]: \left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3(2-4)} = -1/2$$

[c]: $\left.\dfrac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \dfrac{2(4)-5}{(4-3)(4-2)} = \dfrac{3}{2}$

$$Y(s) = \left[\frac{-1}{s-3}\right] - \left[\frac{1}{2}\cdot\frac{1}{s-2}\right] + \left[\frac{3}{2}\cdot\frac{1}{s-4}\right]$$

$$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$ //