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15/ENG03/011

CIVIL ENGINEERING

ENG 381

ASSIGNMENT 3

1) $y = e^{x^2+x}$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$$

$$y'' = y'(2x+1) + 2y$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'(2x+1) = (2x+1)e^{x^2+x}(2x+1) \\ = (2x+1)^2 e^{x^2+x}$$

$$2y = 2 \cdot e^{x^2+x}$$

$$\therefore y''(2x+1) + 2y = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$$

$$\therefore y'' = y'(2x+1) + 2y$$

$$\therefore 2e^{x^2+x} + (2x+1)^2 e^{x^2+x} = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$$

$$w = y''$$

$$w' = y^{n+2}$$

$$P = y'(2x+1)$$

$$V = 2x+1$$

$$u = y'$$

$$V' = 2$$

$$u' = y^{n+1}$$

$$V'' = 0$$

$$P^n = y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2$$

$$S = 2y$$

$$S^n = 2y^n$$

$$W^n = P^n + S^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$$

2) Using the Leibnitz theorem, given that

i) $y = x^3 e^{4x}$ determine y^5

$V = x^3$

$U = e^{4x}$

$V' = 3x^2$

$U' = 4e^{4x}$

$V'' = 6x$

$U'' = 4 \cdot 4 e^{4x}$

$V''' = 6$

$U''' = 4 \cdot 4 \cdot 4 e^{4x}$

$V^{(4)} = 0$

$U^{(4)} = 4^4 e^{4x}$

$$y^n = 4^n e^{4x} x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x$$

$$+ \frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} \cdot 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x$$

$$+ \frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} \cdot 6$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \cdot 4 e^{4x} 3x^2 + 5 \cdot 4 \cdot 4^3 e^{4x} 3x$$

$$+ 5 \cdot 4 \cdot 3 \cdot 4^2 e^{4x}$$

iii) $x^2 y'' + x y' + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$

Let $w = x^2 y''$

$V = x^2$

$U = y''$

$V' = 2x$

$U' = y'''$

$V'' = 2$

$U'' = y^{(4)}$

$V^{(3)} = 0$

$U^{(n)} = y^{(n+2)}$

$$w^n = y^{(n+2)} x^2 + n \cdot y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2} \cdot y^n \cdot 2$$

$$w^n = x^2 y^{(n+2)} + n 2x y^{(n+1)} + n(n-1) y^n$$

Let $p = x y'$

$V = x$

$U = y'$

$V' = 1$

$U' = y''$

$V'' = 0$

$U'' = y^{(n+1)}$

$$P^n = y^{n+1} \cdot x + n \cdot y^n - 1$$

$$P^n = x y^{n+1} + n y^n$$

$$S = y$$

$$S^n = y^n$$

$$y^{n+2} \cdot x^2 + n \cdot 2xy^{n+1} + n(n-1)y^n + y^{n+1}x + ny^n + y^n = 0$$

$$y^{n+2} x^2 + (n \cdot 2xy^{n+1} + xy^{n+1}) + [n(n-1)y^n + ny^n + y^n] = 0$$

$$x^2 y^{n+2} + (2n+1)xy^{n+1} + [n(n-1) + 1 + n]y^n = 0$$

$$x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2 + 1)y^n = 0$$