

SMOOTH SIMON

IS/ENR03/030

CIVIL ENGINEERING

ASSIGNMENT 5

i) $\frac{dy}{dt} + 3y = e^{-2t}$ given at $t=0$ $y=2$
 $y(s) = 2$

$$y^{(1)} + 3y^{(0)} = e^{-2t}$$

$$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$$

$$sY(s) - 2 + 3Y(s) = \frac{1}{s+2}$$

$$Y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1} = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2}$$

$$Y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

A: $\frac{1+2s+4}{s+3} \Big|_{s=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

B: $\frac{1+2s+4}{s+3} \Big|_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

$$Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y(t) = e^{-2t} + e^{-3t}$$

ii) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$
 $y(s) = 1$

$$3y^{(1)} - 6y = \sin 2t$$

$$3(sY(s) - y(0)) - 6Y(s) = \frac{2}{s^2+4}$$

$$3sY(s) - 3Y(s) - 6Y(s) = \frac{2}{s^2+4}$$

$$Y(s)[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$$

$$Y(s)[3s-6] = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+4)}$$

$$y(s) = \frac{3s^2 + 14}{(s^2 + 4)(3s - 6)} = \frac{A}{(s^2 + 4)} + \frac{C}{(3s - 6)}$$

$$C: \frac{3s^2 + 14}{s^2 + 4} \Big|_{s=2} = \frac{3(2)^2 + 14}{2^2 + 4} = \frac{13}{4}$$

$$3s^2 + 14 = (A + Bs)(3s - 6) + C(s^2 + 4)$$

$$3s^2 + 14 = 3As - 6A + 3Bs^2 - 6Bs + (s^2 + 4C)$$

Comparing coefficients

$$3 = 3B + C$$

$$3 = 3B + \frac{13}{4} \quad \left(\text{where } C = \frac{13}{4} \right)$$

$$3A - 6B = 0$$

$$3A = 6B$$

$$3A = 6 \times -1 \quad A = -1/6$$

$$y(s) = \frac{-1/6}{s^2 + 4} - \frac{1/12s}{3s - 6} + \frac{13/4}{3s - 6}$$

$$\frac{-1/6}{s^2 + 4} - \frac{1/12s}{s^2 + 4} + \frac{13/4}{3s - 6}$$

$$= \frac{-1}{6} \cdot \frac{1}{s^2 + 2^2} - \frac{1}{12} \frac{s}{s^2 + 2^2} + \frac{13}{4} \cdot \frac{1}{3(s - 2)}$$

$$= \frac{-1}{6} \cdot \frac{1}{2} \left[\frac{2}{s^2 + 2^2} \right] - \frac{1}{12} \left[\frac{s}{s^2 + 2^2} \right] + \frac{13}{12} \left[\frac{1}{s - 2} \right]$$

$$y(t) = \frac{-1}{12} \sin 2t - \frac{1}{12} \cos 2t + \frac{13}{12} e^{2t}$$

$$y(t) = \frac{1}{12} [-\sin 2t - \cos 2t + 13e^{2t}]$$

$$y(t) = \frac{1}{12} [13e^{2t} - \cos 2t - \sin 2t]$$

3) $\frac{dy}{dt} - 4y = 8$ given that $t=0$ $y=2$ $y(0)=2$

$$y'(t) - 4y = 8$$

$$sY(s) - y(0) - 4Y(s) = \frac{8}{s}$$

$$sY(s) - 2 - 4Y(s) = \frac{8}{s}$$

$$y(s) [s-4] = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$A: \frac{8+2s}{s-4} \Big|_{s=0} = \frac{8}{-4} = -2$$

$$B: \frac{8+2s}{s} \Big|_{s=4} = \frac{8+2(4)}{4} = 4$$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

4) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}$ at $t=0, y=2, y''=1$

$$y^{(2)} - 2y^{(1)} + 5y = e^{2x}$$

$$(s^2 y(s) - s y'(0) - y''(0)) - 2(s y(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2s y(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$y(s) [s^2 - 2s + 5] = \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1} = \frac{1+2s(s-2)-3(s-2)}{s-2}$$

$$y(s) [s^2 - 2s + 5] = \frac{1+2s^2-4s-3s+6}{s-2} = \frac{2s^2-7s+7}{s-2}$$

$$y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$$

$$y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$$

$$A: \frac{2s^2-7s+7}{s^2-2s-5} \Big|_{s=-2} = \frac{2(-2)^2-7(-2)+7}{(-2)^2-2(-2)+5} = \frac{1}{5}$$

$$2s^2-7s+7 = A(s^2-2s+5) + (Bs+C)(s-2)$$

$$2s^2-7s+7 = As^2-2As+5A+Bs^2-2Bs+Cs-2C$$

$$2 = A+B$$

$$2 = \frac{1}{5} + B \Rightarrow B = \frac{9}{5}$$

$$7 = 5A - 2C$$

$$7 = 5\left(\frac{1}{5}\right) - 2C$$

$$7 - 1 = -2C$$

$$C = -3$$

$$y(s) = \frac{1}{s-2} + \frac{9s-3}{s^2-2s+5} = \frac{1}{s-2} + \frac{1}{s-2} + \frac{9s}{(s+1)^2+4} - \frac{3}{(s+1)^2}$$

$$y(s) = \frac{1}{s} + \frac{1}{s-2} + \frac{9}{5} \frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$y(s) = \frac{1}{s} + \frac{1}{s-2} + \frac{9}{5} \frac{(s+1)}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$y(s) = \frac{1}{s} + \frac{1}{s-2} + \frac{9}{5} \frac{s+1}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2} \cdot \frac{2}{2}$$

$$y(t) = \frac{1}{5} e^{2t} + \frac{9}{5} e^{-t} (\cos 2t - 2e^{-t} \sin 2t)$$

$$y(t) = \frac{1}{5} \left[e^{2t} + \frac{9}{5} e^{-t} (\cos 2t - 10e^{-t} \sin 2t) \right]$$

$$= \frac{1}{5} \left[e^{2t} + e^{-t} (9 \cos 2t - 10 \sin 2t) \right]$$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$ $y=0$ $y'(0)=2$

$$y^{(1)} - 6y^{(0)} + 8y = e^{3t}$$

$$s^2 y(s) - 6[sy(s) - y(0)] + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$y(s) [s^2 - 6s + 8] = \frac{1}{s-3} + \frac{2}{1} = \frac{1+2(s-3)}{s-3} = \frac{1+2s-6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$A: \frac{2s-5}{(s-2)(s-4)} \Big|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$$

$$B: \frac{2s-5}{(s-2)(s-4)} \Big|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = -\frac{1}{2}$$

$$C: \frac{2s-5}{(s-3)(s-2)} \Big|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$$

$$y(s) = \frac{-1}{s-3} - \frac{1}{2} \cdot \frac{1}{s-2} + \frac{3}{2} \cdot \frac{1}{s-4}$$

$$y(t) = -e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$