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15/ENG07/032

Petroleum Engineering

1. $e^{x^2+x} = y$

$y' = [2x+1]\, e^{x^2+x}$

using Product rule

$y'' = [2x+1][2x+1]e^{x^2+x} + 2[e^{x^2+x}]$

Since y' & y are in y''
Substitute

$y'' = [2x+1]\, y' + 2y$

$y'' = y'[2x+1] + 2y$

For y''

$u = y''$, $u' = y'''$, $u^n = y^{n+2}$

$v = 1$ $\quad v' = 0$

Note $v = v_0 = 1$

$\therefore = u^n v^0 + n \cdot u^{n-1} v^1$

$= y^{n+2} \cdot 1 + \cancel{n y^{n+1} \cdot 0}$

$= y^{n+2}$

For $y'[2x+1]$

$u = y'$, $u' = y''$, $u^n = y^{n+1}$

$v^0 = v = [2x+1]$, $v' = 2$, $v'' = 0$

$= u^n v^0 + n \cdot u^{n-1} \cdot v' + \dfrac{n[n-1] u^{n-2} v''}{2!}$

$= y^{n+1}[2x+1] + n[y^n] \cdot 2 + \cancel{\dfrac{n[n-1] y^{n-1} \cdot 0}{2!}}$

$= [2x+1] y^{n+1} + 2n y^n$

For $2y$

$u^n = y^n$ $\quad \therefore = 2y^n$

Combining them

$\Rightarrow y^{n+2} = [2x+1]y^{n+1} + 2ny^{n} + 2y^{n}$

$y^{n+2} = [2x+1]y^{n+1} + 2y^{n}[n+1]$

2i $y = x^3 e^{4x}$ determine $y^{[5]}$

Let $u = x^3$

$u' = 3x^2$

$u'' = 6x$

$u''' = 6$

$u^{iv} = 0$

$u^{v} = 0$

Let $v_1 = e^{4x} = v_0$

$v' = 4e^{4x}$

$v'' = 16e^{4x}$

$v''' = 64e^{4x}$

$v^{iv} = 256e^{4x}$

$v^{v} = 1024e^{4x}$

$y^{[5]} = u^{n}v^{0} + nu^{n-1}v' + \frac{n[n-1]u^{n-2}v''}{2!}$

$$+\frac{n[n-1][n-2]\,u^{n-3}\,v'''}{3!}+\frac{n[n-1][n-2][n-3]\,u^{n-4}\,v^{iv}}{4!}$$

$$+\frac{n[n-1][n-2][n-3][n-4]\,u^{n-5}\,v^{v}}{5!}$$

$$y^5 = x^3 \cdot e^{4x} + n \cdot 3x^2 \cdot 4e^{4x} + \frac{n[n-1] \cdot 6x \cdot 16e^{4x}}{2\times 1} +$$

$$\frac{n[n-1][n-2] * 6 * 64e^{4x}}{3\times 2\times 1} + \cancel{\frac{n[n-1][n-2][n-3] * 0 * 256e^{4x}}{4\times 3\times 2\times 1}}$$

$$+\cancel{\frac{n[n-1][n-2][n-3][n-4] * 0 * 1024e^{4x}}{5\times 4\times 3\times 2\times 1}}$$

$$y^{[5]} = x^3e^{4x} + 12n\,x^2e^{4x} + 48n[n-1]x\,e^{4x} +$$
$$64n[n-1][n-2]\,e^{4x}$$

$$y^5 = x^3e^{4x} + 12\times 5x^2e^{4x} + 48\times 5[5-1]x e^{4x} +$$
$$64\times 5[5-1][5-2]\,e^{4x}$$

$$y^5 = x^3e^{4x} + 60x^2e^{4x} + 960xe^{4x} + 3840e^{4x}$$

ii $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

For x^2y''

Let $u = y''$, $u' = y'''$, $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$, $v'' = 2$, $v''' = 0$

$= u^n v^0 + n u^{n-1} v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!}$

$= y^{n+2}\cdot x^2 + n y^{n+1}\cdot 2x + \frac{n(n-1)y^n\cdot \cancel{2}}{\cancel{2\times 1}} + \cancel{\frac{n(n-1)(n-2)y^{n-1}\cdot 0}{3\times 2\times 1}}$

$= x^2y^{n+2} + 2xny^{n+1} + n(n-1)y^n$

For xy'

Let $u = y'$, $u' = y''$, $u^n = y^{n+1}$

$v = x$, $v' = 1$, $v'' = 0$

$$= u^n v^0 + n[u^{n-1}]v' + \frac{n[n-1]u^{n-2}v''}{2!} + \ldots$$

$$= y^{n+1}\cdot x + n\cdot y^n\cdot 1 + \cancel{\frac{n[n-1]y^{n-1}\cdot 0}{2!}}$$

$$= xy^{n+1} + ny^n$$

For y

$$y^n = y^n$$

Combining

$$= x^2y^{n+2} + 2xny^{n+1} + n[n-1]y^n + xy^{n+1} + ny^n + y^n$$

$$\Rightarrow x^2y^{n+2} + 2x[n+1]y^{n+1} + [n^2+1+n-n]y^n$$

$$\therefore \Rightarrow x^2y^{n+2} + 2x[n+1]y^{n+1} + [n^2+1]y^n = 0$$