15/ENG07/032

Petroleum Engineering 300L

$$\frac{d^2y}{d\theta^2} + \frac{4dy}{d\theta} + 5y = 6\sin\theta$$

$$y = e^{K\theta}$$
$$y^1 = Ke^{K\theta}$$
$$y^{11} = K^2 e^{K\theta}$$

$$y'' + 4y' + 5y = 6\sin\theta$$

for Homogeneity

$$K^2 + 4K + 5 = 0$$

using Completing the Square method

$$K^2 + 4K + 2^2 = -5 + 2^2$$
$$(K+2)^2 = -1$$
$$K + 2 = \pm i$$
$$K = -2 \pm i$$
$$K_1 = -2 + i \qquad K_2 = -2 - i$$

$$y = C_1 y_1 + C_2 y_2$$
$$y = C_1 \cdot e^{-2\theta} \cdot e^{i\theta} + C_2 \cdot e^{-2\theta} \cdot e^{-i\theta}$$

$$y_p = e^{-2\theta}[C_1 \cos\theta + C_2 \sin\theta]$$

For Non-Homogeneity

Let
$$y = A\sin\theta + B\cos\theta$$
$$y' = A\cos\theta - B\sin\theta$$
$$y'' = -A\sin\theta - B\cos\theta$$

$$-A\sin\theta - B\cos\theta + 4A\cos\theta - 4B\sin\theta + 5A\sin\theta + 5B\cos\theta = 6\sin\theta + 0\cos\theta$$

$$-A - 4B + 5A = 6 \Rightarrow 4A - 4B = 6 \quad \text{(2)}$$
$$-B + 4A + 5B = 0 \Rightarrow -\ 4A + 4B = 0$$
$$-8B = 6$$
$$B = -3/4$$

Scanned by CamScanner

Put $B = -3/4$ into ②

$4A - 4B = 6$

$4A - \cancel{4}[-3/\cancel{4}] = 6$

$4A + 3 = 6$

$4A = 6 - 3$

$A = 3/4$

$\therefore y_N = 3/4 \sin\theta - \frac{3}{4}\cos\theta$

In general

$y = y_N + y_P$

$= e^{-2\theta}[C_1\cos\theta + C_2\sin\theta] + 3/4\sin\theta - 3/4\cos\theta$

iii) $y = \frac{3}{4}\sin\theta - \frac{3}{4}\cos\theta$

$y' = \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta = 0$

$\frac{3}{4}\cos\theta = -\frac{3}{4}\sin\theta$

Divide through by $\cos\theta$

$\frac{3}{4} = \frac{-3}{4}\frac{\sin\theta}{\cos\theta}$

$\frac{3}{4} = \frac{-3}{4}\tan\theta$

$\theta = \tan^{-1}\left[\frac{3 \times 4}{-3 \times 4}\right]$

$\theta = -45^\circ$

Scanned by CamScanner

Excellent ·
KUNAMI KUTA TAKU TO CHI

$$EI\frac{d^2y}{dx^2} = \frac{w}{2}[L-x]^2$$

$$y'' = \frac{w}{2EI}[L-x]^2$$

Homogeneity

$$y = e^{kx}$$
$$y' = ke^{kx}$$
$$y'' = k^2e^{kx}$$

$$y'' + 4y' + 8y = 0$$
$$y = e^{kx}$$

$$k^2e^{kx} = 0$$
$$k^2 = 0$$
$$k = 0 \quad [\text{twice}]$$

KARIN KATO TE

$$y_h = [Ax + B]e^{kx}$$
$$= [Ax + B]e^{0x} = Ax + B$$

Non-Homogeneity

$$y'' = \frac{w}{2EI}[L^2 - 2Lx + x^2] = \frac{wL^2}{2EI} - \frac{wLx}{EI} + \frac{wx^2}{2EI}$$

$$y_p = C + Dx + fx^2$$

But there are solutions in terms of x at order zero & one from the homogeneous solution,

$\therefore \quad y_p = [C + Dx + Fx^2]x^2$

$y_p = Cx^2 + Dx^3 + Fx^4$

$y_p' = 2Cx + 3Dx^2 + 4Fx^3$

$y_p'' = 2C + 6Dx + 12Fx^2$

To solve for C, D, & F

$$2C + 6Dx + 12Fx^2 = \frac{wl^2}{2EI} - \frac{wlx}{EI} + \frac{wx^2}{2EI}$$

Comparing

$$\frac{wl^2}{2EI} = 2C$$

$$C = \frac{wl^2}{4EI} \quad \text{—— } (1)$$

$$6D = -\frac{wl}{EI}$$

$$D = -\frac{wl}{6EI} \quad \text{—— } (2)$$

$$12F = \frac{w}{2EI}$$

$$F = \frac{w}{24EI}$$

$$\therefore\ y_p = \left[\frac{wL^2}{4EI} - \frac{wLx}{6EI} + \frac{wx^2}{24EI}\right]x^2$$

$$y = y_h + y_p$$

$$y = Ax + B + \left[\frac{wl^2}{4EI} - \frac{wLx}{6EI} + \frac{wx^2}{24EI}\right]x^2$$

$$y = Ax + B + \frac{wl^2x^2}{4EI} - \frac{wlx^3}{6EI} + \frac{wx^4}{24EI}\Big]$$

$x = 0$, $y = 0$

$$0 = A[0] + B + \frac{wl^2[0]}{4EI} - \frac{wL[0]^3}{6EI} + \frac{w[0]}{24EI}$$

$$B = 0$$

$$y' = A + \frac{wl^2x}{2EI} - \frac{wlx^2}{2EI} + \frac{wx^3}{6EI}$$

$x = 0$, $y' = 0$

$\therefore A = 0$

$$y = Ax + B + \frac{wl^2x^2}{4EI} - \frac{wlx^3}{6EI} + \frac{wx^4}{24EI}$$

$$y = \frac{wl^2x^2}{4EI} - \frac{wlx^3}{6EI} + \frac{wx^4}{24EI}$$

$$y = \frac{w}{24EI}\left[6l^2x^2 - 4lx^3 + x^4\right]$$

$$y = \frac{w}{24EI}\left[6l^2 - 4lx + x^2\right]x^2$$

$x = l$

$$y = \frac{w}{24EI}\left[6l^2 - 4l \times l + l^2\right]l^2$$

$$= \frac{w}{\cancel{24}_{8}EI} \cdot [\cancel{3}^{1}l^2]\,l^2 = \frac{wl^4}{8EI}$$

Scanned by CamScanner