Ogbewele Excellent O
15/ENG07/032
PETROLEUM 300L

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$$y'' - y' - 2y = 8$$

For Homogeneity

Let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

$k^2 e^{kx} - ke^{kx} - 2e^{kx} = 0$

$\underbrace{k^2 - k - 2}_{} = 0$ [Since $e^{kx} = y$]

$-2k^2 = -2k$ \$ k

$[k-2][k+1] = 0$

$k_1 = 2$ \$ $k_2 = -1$

$y_1 = e^{k_1 x} = e^{2x}$

$y_2 = e^{k_2 x} = e^{-x}$

$y_p = C_1 y_1 + C_2 y_2 = C_1 e^{2x} + C_2 e^{-x}$

For Non-Homogeneity

Let $y = A$

$y' = 0$

$y'' = 0$

$\therefore -2A = 8$

$A = -4$

$y_N = -4$

In general

$y = y_p + y_N$

$y = C_1 e^{2x} + C_2 e^{-x} - 4$

2. $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

For Homogeneity

Let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2 - 4 = 0$

$k^2 = 4$

$k = \pm 2$

$y_1 = e^{k_1 x} = e^{2x}$

$y_2 = e^{k_2 x} = e^{-2x}$

$y_P = e^{kx}[C_1 + xC_2]$

$= e^{2x}[C_1 + xC_2]$

For Non-Homogeneity

Let $y = Ae^{3x}$

$y' = 3Ae^{3x}$

$y'' = 9Ae^{3x}$

$[9A - 4A]\cancel{e^{3x}} = 10\cancel{e^{3x}}$

$5A = 10$

$A = 2$

$y_N = 2e^{3x}$

In general

$y = y_P + y_N$

$= e^{2x}[C_1 + xC_2] + 2e^{3x}$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$$

$$y'' + 2y' + y = e^{-2x}$$

For Homogeneity

Let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2 + 2k + 1 = 0$

$k^2 = k$ & k

$[k+1][k+1] = 0$

$k = -1$

$y = e^{kx}[C_1 + xC_2]$

$y_p = e^{-x}[C_1 + xC_2]$

For Non-Homogeneity

Let $y = Ae^{-2x}$

$y' = -2e^{-2x}A$

$y'' = 4e^{-2x}A$

$[4 + 2[-2] + 1] \cdot e^{-2x}A = e^{-2x}$

$A = 1$

$\therefore y_N = e^{-2x}$

In general

$y = y_p + y_N$

$$y = e^{-x}[C_1 + xC_2] + e^{-2x}$$

$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$y'' + 25y = 5x^2 + x$$

For Homogeneity

Let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$$k^2 + 25 = 0$$

$$k^2 = -25$$

$$k = \pm 5i$$

$y_1 = e^{5ix}$

$y_2 = e^{-5ix}$

$$y = C_1y_1 + C_2y_2$$

$$y = C_1e^{5ix} + C_2e^{-5ix}$$

$$y_p = C_1\cos 5x + C_2\sin 5x$$

For Non - Homogeneity

Let $y = Ax^2 + Bx + C$

$y' = 2Ax + B$

$y'' = 2A$

$$2A + 25[Ax^2 + Bx + C] = 5x^2 + x$$

$$25Ax^2 + 25Bx + 2A + 25C = 5x^2 + x$$

Comparing both sides

$$25A = 5$$

$$25B = 1$$

$$2A + 25C = 0$$

$$A = 1/5$$

$$B = 1/25$$

$$2[1/5] + 25C = 0$$

$$25C = -2/5 \Rightarrow C = -2/125$$

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5. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$y'' - 2y' + y = 4\sin x$

For Homogeneity

Let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$\underbrace{k^2 - 2k + 1 = 0}$

$1k^2 = -k \ \$ \ -k$

$[k-1][k-1] = 0$

$k = 1$

$y_p = e^{kx}[C_1 + xC_2]$

$= e^{x}[C_1 + xC_2]$

For Non-Homogeneity

Let $y = A\sin x + B\cos x$

$y' = A\cos x - B\sin x$

$y'' = -A\sin x - B\cos x$

$-A\sin x - B\cos x - 2[A\cos x - B\sin x] + A\sin x + B\cos x = 4\sin x + 0\cos x$

$[-A + 2B + A]\sin x + [-B - 2A + B]\cos x = 4\sin x + 0\cos x$

Comparing b.s

$B\sin x - A\cos x = 4\sin x + 0\cos x$

$\therefore 2B = 4 \qquad B = 2$

$-2A = 0$

$\therefore A = 0 \qquad \Rightarrow y_N = 2\cos x$

In general $y = y_P + y_N$

$y = e^{x}[C_1 + xC_2] + 2\cos x$

6. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$y'' + 4y' + 5y = 2e^{-2x}$

Homogeneity Let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2 + 4k + 5 = 0$

using Completing the square method

$k^2 + 4k + [2]^2 = -5 + 2^2$

$[k+2]^2 = -1$

$k + 2 = \pm i$

$k = -2 \pm i$

$k_1 = -2 + i$

$k_2 = -2 - i$

$y = C_1 y_1 + C_2 y_2$

$y = C_1 \cdot e^{-2x} \cdot e^{ix} + C_2 \cdot e^{-2x} e^{-ix}$

$y_p = e^{-2x} [C_1 \cos x + C_2 \sin x]$

For Non-Homogeneity

Let $y = Axe^{-2x}$

$y' = A[e^{-2x} - 2xe^{-2x}]$

$y'' = A[-2e^{-2x} - 2[e^{-2x} - 2xe^{-2x}]$

$= A[-2e^{-2x} - 2e^{-2x} + 4xe^{-2x}]$

$= A[-4e^{-2x} + 4xe^{-2x}]$

$A[-4e^{-2x} + 4xe^{-2x}] + 4A[e^{-2x} - 2xe^{-2x}] + 5Axe^{-2x}$

$= 2e^{-2x}$

$[4Ax - 4A]e^{-2x} + [4A - 8Ax]e^{-2x} + [5Ax]e^{-2x} = 2e^{-2x}$

$5Ax - 8Ax + 4Ax = 2$

$Ax = 2$

$y_N = 2 \cdot e^{-2x}$

7.

OR

Let $y = Ae^{-2x}$

$y' = -2Ae^{-2x}$

$y'' = 4Ae^{-2x}$

$4A - 8A + 5A = 2$

$A = 2$

$\therefore \; y_N = 2e^{-2x}$

In general $y = y_P + y_N$

$y = e^{-2x}[C_1 \cos x + C_2 \sin x] + 2e^{-2x}$

$$3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$$

$3y'' - 2y' - y = 2x - 3$

Homogeneity

Let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2 e^{kx}$

$3k^2 - 2k - 1 = 0$

$-3k^2 = -3k$ \$ k

$[3k^2 - 3k] + [k - 1] = 0$

$3k[k - 1] + 1[k - 1] = 0$

$[3k + 1][k - 1] = 0$

$k_1 = -1/3 \qquad k_2 = 1$

$y_P = C_1 y_1 + C_2 y_2$

$= C_1 \cdot e^{-1/3 x} + C_2 e^{x}$

For Non - Homogeneity

Let $y = Ax + B$

$y' = A$

$y'' = 0$

$-2A - Ax - B = 2x - 3$

Comparing both sides

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$-2A - B = 3$
$-A = 2$
$\therefore A = -2$

$-2[-2] - B = 3$
$4 - B = 3$
$B = 1$

$y_N = -2x + 1$

In general
$y = y_P + y_N$
$= c_1 \cdot e^{-1/3x} + C_2 \cdot e^{x} - 2x + 1$

8. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$y'' - 6y' + 8y = 8e^{4x}$
Homogeneity
$y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$\underbrace{k^2 - 6k + 8}_{8k^2 = -4k \ \& \ -2k} = 0$

$[k-4][k-2] = 0$
$k_1 = 4 \quad \& \quad k_2 = 2$

$y = C_1 y_1 + C_2 y_2$
$y_P = C_1 e^{4x} + C_2 e^{2x}$

For Non-Homogeneity
$y = Ae^{4x}$
$y' = 4Ae^{4x}$
$y'' = 16Ae^{4x}$

OR

CONTINUATION

4 In general

$y_N = \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

$\therefore y = y_P + y_N$

$y = C_1 \cos 5x + C_2 \sin 5x + \frac{1}{5}x^2 + \frac{1}{25}x - \frac{2}{125}$

6. given that $x = 0$, $y = 1$, $y' = -2$

$y = C_1 e^{-2x} \cos x + C_2 e^{-2x} \sin x + 2e^{-2x}$

$y' = C_1[-2e^{-2x}\cos x - e^{-2x}\sin x] + C_2[-2e^{-2x}\sin x + e^{-2x}\cos x]$
$-4e^{-2x}$

$1 = C_1 + 2$ — ⊛
$-2 = C_1 + C_2 - 4$

$1 - 2 = C_1$

$C_1 = -1$

Put $-1 = C_1$ — ⊛

$1 = -1 + C_2 + 2$

$C_2 = 0$

$\therefore y = -e^{-2x}\cos x + 2e^{-2x}$

8. $y = Axe^{4x}$

$y' = A[e^{4x} + 4xe^{4x}]$

$y'' = A[4e^{4x} + 4[e^{4x} + 4xe^{4x}]]$

$= A[4e^{4x} + 4e^{4x} + 16xe^{4x}]$

$4A + 4A + 16Ax - 6A - 24Ax + 8Ax = 8$

$2A = 8$

$A = 4$

$y = 4xe^{4x}$

$\therefore$ In general

$y = y_P + y_N$

$y = C_1e^{4x} + C_2e^{2x} + 4xe^{4x}$

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