Assignment V

① $\frac{dy}{dt} + 3y = e^{-2t}$

given that $t=0$, $y=2$

solution

$y'(t) + 3y(t) = e^{-2t}$ ——— (1)

transforming eqn (1) to the s domain

$sy(s) + y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$= \frac{A}{s+3} + \frac{B}{s+2} = \frac{2s+5}{(s+3)(s+2)}$

$A|_{s=-3} = \frac{2(-3)+5}{(-3+2)} = 1$

$A = 1$

$B|_{s=-2} = \frac{2(-2)+5}{(-2+3)} = 1$

$\therefore A = 1 \;\&\; B = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = L^{-1}(y(s))$

$y(t) = e^{-2t} + e^{-3t}$ //

(2) $\frac{dy}{dt} - 4y = 8$

at $t=0, y=2$,

$y'(t) - 4y(t) = 8$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 2 - 4y(s) = 8/s$

$y(s)(s-4) = 8/s + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)} \Rightarrow \frac{A}{s} + \frac{B}{s-4}$

$A|_{s=0} = \frac{8+2(0)}{(0-4)} = \frac{8}{-4} = -2$

$B|_{s=4} = \frac{8+2(4)}{4} = \frac{16}{4} = 4$

$A = -2$ and $B = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = \mathcal{L}^{-1}(y(s))$

$y(t) = -2 + 4e^{4t}$ //

(3) $\frac{dy}{dt} - 4y = 8$ $\quad t=0, y=2$

$y'(t) - 4y(t) = 8$

$sY(s) - y(0) - 4Y(s) = 8/s$

$Y(s)(s-4) - 2 = 8/s$

$Y(s) = \frac{8}{s} + 2 / s-4$

$Y(s) = \frac{8+2s}{s(s-4)} \Rightarrow \frac{A}{s} + \frac{B}{s-4}$

$A|_{s=0} = \frac{8+0}{(0-4)} = \frac{8}{-4} = -2$

$B|_{s=4} = \frac{8+8}{4} = \frac{16}{4} = 4$

$Y(s) = -\frac{2}{s} + \frac{4}{s-4}$

$y(t) = L^{-1}(Y(s))$

$y(t) = -2 + 4e^{4t}$

$y(t) = 4e^{4t} - 2$ //

5 $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0$, $y=0$, $y'=2$

$y''(t) - 6y'(t) + 8y(t) = e^{3t}$

$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 0 - 2 - 6(sy(s) - 0) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$

$y(s)(s^2 - 6s + 8) = \frac{1 + 2s - 6}{s-3} \Rightarrow \frac{2s-5}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$

$y(s) = \frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$

$A|_{s=3} = \frac{2(3)-5}{(3-4)(3-2)} = -1$

$B|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$

$C|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = \frac{-1}{2}$

$A = -1$, $B = 3/2$ and $C = -1/2$

$y(s) = -\frac{1}{s-3} + \frac{3}{2(s-4)} - \frac{1}{2(s-2)}$

$y(t) = \mathcal{L}^{-1}[y(s)]$

$y(t) = -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$ //