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Assignment 1

(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solution

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

Assuming homogeneity

$y'' - y - 2y = 0$

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

Substituting into the question

$k^2e^{kx} - ke^{kx} - 2e^{kx} = 0$

$(k^2 - k - 2)e^{kx} = 0$

$k^2 - k - 2 = 0$

$k^2 + k - 2k - 2 = 0$

$k(k+1) - 2(k+1) = 0$

$(k-2)(k+1) = 0$

$k - 2 = 0,\ k + 1 = 0$

$k = 2,\ k = -1$

$k_1 = 2,\ k_2 = -1$

$y_1 = e^{k_1 x} = e^{2x}$

$y_2 = e^{k_2 x} = e^{-x}$

$y_h = C_1 y_1 + C_2 y_2$

$\therefore y_h = C_1 e^{2x} + C_2 e^{-x}$

Dealing with the non-homogeneous part

$y_P = C$

$\therefore y_P' = 0$

$y_P'' = 0$

Recall; $y_P'' - y_P' - 2y_P = 8$

$0 - 0 - 2C = 8$

$-2C = 8$

$C = -4$

$\therefore y_P = -4$

$\therefore y = y_{homogeneous} + y_{particular}$

$y = C_1 e^{2x} + C_2 e^{-x} - 4$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

Solution

Assuming homogenaity

$y'' - 4y = 0$

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

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$k^2e^{kx} - 4e^{kx} = 0$

$(k^2 - 4)e^{kx} = 0$

$k^2 - 4 = 0$

$k^2 = 4$

$k = \sqrt{4}$

$k = \pm 2$

$k_1 = 2, \quad k_2 = -2$

$y_1 = e^{k_1 x} = e^{2x}$

$y_2 = e^{k_2 x} = e^{-2x}$

$y_{homogeneous} = C_1e^{2x} + C_2e^{-2x}$

$y_{particular}, \ y_p = Ae^{3x}$

$y_p' = 3Ae^{3x}$

$y_p'' = 9Ae^{3x}$

Recall; $y_p'' - 4y = 10e^{3x}$

$9Ae^{3x} - 4Ae^{3x} = 10e^{3x}$

$5Ae^{3x} = 10e^{3x}$

$5A = 10$

$5A/5 = 10/5$

$A = 2$

$y_p = 2e^{3x}$

$y = y_h + y_p$

$= C_1e^{2x} + C_2e^{-2x} + 2e^{3x}$

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③ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

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Solution

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

Assuming homogeneity

$y'' + 2y' + y = 0$

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

Substituting into the question

$k^2e^{kx} + 2ke^{kx} + 1e^{kx} = 0$

$(k^2 + 2k + 1)e^{kx} = 0$

$k^2 + 2k + 1 = 0$

$k^2 + k + k + 1 = 0$

$k(k+1) + 1(k+1) = 0$

$(k+1)(k+1) = 0$

$k+1 = 0, \quad k+1 = 0$

$k = -1, \quad k = -1$

$k = -1, \quad k = -1$

$k = -1$ twice

$y = e^{kx} = e^{-x}$ twice

$y_{homogeneous} = c_1y_1 + xc_2y_2$

$= c_1e^{-x} + xc_2e^{-x}$

Dealing with the non-homogenous part

$y_{non-homogenous}$ ~~Part~~ $= Ae^{-2x}$

$y_{non-homogenous} = Ae^{-2x}$

$y_p' = -2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

Recall; $y_p'' - 2y_p'' + y_p = e^{-2x}$

$4Ae^{-2x} + 2(-2Ae^{-2x}) + Ae^{-2x} = e^{-2x}$

$4Ae^{-2x} - 4Ae^{-2x} + Ae^{-2x} = e^{-2x}$

$Ae^{-2x} = e^{-2x}$

$A = 1$

$y_p = Ae^{-2x} = e^{-2x}$

$y_s = y_h + y_p$

$= e^{-x}(c_1 + xc_2) + e^{-2x}$

(4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

Solution

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

Assuming homogeniety

$y'' + 25y = 0$

$k^2e^{kx} + 25e^{kx} = 0$

$k^2 + 25 = 0$

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$k^2 = -25$

$k = \sqrt{-25}$

$k = \pm 5i$

$k_1 = 5i$, $k_2 = -5i$

$y_1 = e^{k_1 x} = e^{5ix}$

$y_2 = e^{k_2 x} = e^{-5ix}$

$y_h = C_1 y_1 + C_2 y_2$

$y_h = C_1 e^{5ix} + C_2 e^{-5ix}$

$= e^{x}(C_1 \cos 5x + C_2 \sin 5x)$

Dealing with the non-homogenous Part

$y_p = Ax^2 + Bx + C$

$y'' = 2Ax + B$

$y'' = 2A$

Recall; $y'' + 25y = 5x^2 + x$

$2A + 25(Ax^2 + Bx + C) = 5x^2 + x$

$2A + 25Ax^2 + 25Bx + 25C = 5x^2 + x$

$+25A = 5$

$A = +5/25 = +1/5$

$+25B = 1$

$B = +1/25$

$2A - 25C = 0$

$2A = -25C$

$-2/5 = 25C$

$C = -2/125$

$\therefore y_{non\text{-}homogenous} = +\frac{1}{5}x^2 + \frac{1}{125}x - \frac{2}{125}$

$y = y_p + y_{non\text{-}homogenous}$

$= C_1e^{5x} + C_2e^{-5x} + \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

$= C_1\cos 5x + C_2\sin 5x + \frac{x^2}{5} + \frac{x}{25} - \frac{2}{125}$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

solution

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} - 2ke^{kx} + e^{kx} = 0$

$k^2 - 2k + 1 = 0$

$k^2 - k - k + 1 = 0$

$k(k-1) - 1(k-1) = 0$

$(k-1)(k-1) = 0$

$k-1=0, \; k-1=0$

$k=1, \; k=1$

$k=1$ twice

$y_h = C_1y_1 + xC_2y_2$

$y_1 = y_2 = e^{kx} = e^x$

$y_h = C_1e^x + xC_2e^x$

$= e^x(C_1 + xC_2)$

$y_p = A\sin x + B\cos x$

$y'_p = A\cos x - B\sin x$

$y''_p = -A\sin x - B\cos x$

Recall; $y'' - 2y' + y = 4\sin x$

$-A\sin x - B\cos x - 2(A\cos x - B\sin x) + A\sin x + B\cos x = 4\sin x$

$-A\sin x - B\cos x - 2A\cos x + 2B\sin x + A\sin x + B\cos x = 4\sin x$

$2B = 4$

$B = 4/2$

$B = 2$

$-2A = 0$

$A = 0$

$y_p = 0\sin x + 2\cos x = 2\cos x$

$y = y_h + y_p$

$y = e^{x}(c_1 + xc_2) + 2\cos x$

(5) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5 = 2e^{-2x}$

Solution

Assuming homoganiety

$y'' + 4y' + 5y = 0$

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

$k^2e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$k^2 + 4k + 5 = 0$

$k^2 + 4k = -5$

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$k^2 + 4k + 2^2 = -5 + 2^2$

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$(k+2)^2 = -1$

$k + 2 = \sqrt{-1}$

$k = \pm\sqrt{-1} - 2$

$k = \pm i - 2$

$k_1 = i - 2 \quad , \quad k_2 = -i - 2$

$y_1 = e^{k_1 x} = e^{(i-2)x} = e^{ix} \cdot e^{-2x}$

$y_2 = e^{k_2 x} = e^{(-i-2)x} = e^{-ix} \cdot e^{-2x}$

$y_h = C_1 y_1 + C_2 y_2$

$y_h = (C_1 e^{ix} \cdot e^{-2x}) + (C_2 e^{-ix} \cdot e^{-2x})$

$y_h = e^{-2x}(C_1 e^{ix} + C_2 e^{-ix})$

$y_h = e^{-2x}(C_1 \sin x + C_2 \cos x)$

$y_p = Ae^{-2x}$

$y'_p = -2Ae^{-2x}$

$y''_p = 4Ae^{-2x}$

Recall; $y'' + 4y' + 5y = 2e^{-2x}$

$4Ae^{-2x} + 4(-2Ae^{-2x}) + 5Ae^{-2x} = 2e^{-2x}$

$4Ae^{-2x} - 8Ae^{-2x} + 5Ae^{-2x} = 2e^{-2x}$

$Ae^{-2x} = 2e^{-2x}$

$A = 2$

$\therefore y_p = 2e^{-2x}$

$y = y_{homogenous} + y_{particular}$

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$y_s = y_n + y_p$

$y_s = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

$y_s' = e^{-2x}(-A\sin x + B\cos x) + (-2e^{-2x})(A\cos x + B\sin x) - 4e^{-2x}$

$y_s' = -Ae^{-2x}\sin x + Be^{-2x}\cos x - A2e^{-2x}\cos x - 2Be^{-2x}\sin x - 4e^{-2x}$

$y_s' = Ae^{-2x}(-\sin x - 2\cos x) + Be^{-2x}(\cos x - 2\sin x) - 4e^{-2x}$

Given that $x = 0, y = 1$

$1 = e^{-2(0)}[A\cos(0) + B\sin(0)] + 2e^{-2(0)}$

$1 = 1(A) + 2$

$A = -1$

Given that $dy/dx = -2$ and $x = 0$

$-2 = Ae^{-2(0)}[-\sin(0) - 2(\cos(0))] + Be^{-2(0)}[\cos(0) - 2\sin(0)] - 4e^{-2(0)}$

$-2 = A(-2) + B(1) - 4$

$-2 = -(-2) + B - 4$

$-2 = -(-2) + B - 4$

$-2 = -2 + B$

$B = 0$

$\therefore y_s = e^{-2x}(-\cos x) + 2e^{-2x}$

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⑦ $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

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Solution

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2e^{kx}$

Assuming homogionity

$3y'' - 2y' - y = 0$

$3(k^2e^{kx}) - 2ke^{kx} - e^{kx} = 0$

$3k^2 - 2k - 1 = 0$

$3k^2 - 3k + k - 1 = 0$

$3k(k-1) + 1(k-1) = 0$

$(3k+1)(k-1) = 0$

$3k + 1 = 0$

$k_1 = -1/3$

$k - 1 = 0$

$k_2 = 1$

$k_1 = -1/3, \quad k_2 = 1$

$y_1 = e^{k_1x} = e^{-x/3}$

$y_2 = e^{k_2x} = e^{x}$

$y_h = c_1y_1 + c_2y_2$

$y_h = c_1e^{-x/3} + c_2e^{x}$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

Recall; $3y'' - 2y' - y = 2x - 3$

$3(0) - 2A - Ax - B = 2x - 3$

$-2A - B = -3$ —— ①

$Ax = 2x$

$A = -2$ —— ②

$-2(-2) - B = -3$

$4 + 3 = B$

$B = 7$

$\therefore y_p = -2x + 7$

$y = y_h + y_p$

$y = C_1 e^{-x/3} + C_2 e^{x} - 2x + 7.$

⑧ $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Solution

let $y = e^{kx}$

$y' = ke^{kx}$

$y'' = k^2 e^{kx}$

Assuming homogeniety

$y'' - 6y' + 8y = 0$

$k^2 e^{kx} - 6ke^{kx} + 8e^{kx} = 0$

$k^2 - 6k + 8 = 0$

$k^2 - 4k - 2k + 8 = 0$

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$K(K-4)-2(K-4)=0$

$(K-2)(K-4)=0$

$K-2=0, \quad K-4=0$

$K_1=2, \quad K_2=4$

$y_1=e^{K_1x}=e^{2x}$

$y_2=e^{4x}$

$y_h=C_1y_1+C_2y_2$

$y_h=C_1e^{2x}+C_2e^{4x}$

$y_p=Axe^{4x}$

$y_p'=A[x(4e^{4x})+e^{4x}]$

$=A(4xe^{4x}+e^{4x})=4Axe^{4x}+Ae^{4x}$

$y_p''=A[(4x(4e^{4x})+e^{4x}(4))+4e^{4x}]$

$=A(16xe^{4x}+4e^{4x}+4e^{4x})$

$=16Axe^{4x}+8Ae^{4x}$

Recall; $y''-6y+8y=0$

$8Ae^{4x}+16Axe^{4x}-6(Ae^{4x}+4xe^{4x})+8Axe^{4x}=8e^{4x}$

$2A=8$

$A=4$

$y_p=4Axe^{4x}$

$y_s=y_h+y_p$

$y_s=C_1e^{2x}+C_2e^{4x}+4Axe^{4x}.$