Name : Sam-Boms Fortune

Matric No : 16/ENG07/033

Department : Petroleum Engineering

course code : ENG 381

Assignment 2

1

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 6\sin\theta$$

$y'' + 4y' + 5y = 0$

let $y = e^{kx}$, $y' = ke^{kx}$, $y'' = k^2e^{kx}$

$k^2e^{kx} + 4ke^{kx} + 5e^{kx} = 0$

$e^{kx}(k^2 + 4k + 5) = 0$

where $a = 1, b = 4, c = 5$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$k = -2 \pm i$

$y_h = e^{-2x}(C_1\cos x + C_2\sin\theta)$

let $y = A\cos\theta + B\sin\theta$

$y' = -A\sin\theta + B\cos\theta$

$y'' = -A\cos\theta - B\sin\theta$

$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$\cos\theta(-A + 4B + 5A) + \sin\theta(-B - 4A + 5B) = 6\sin\theta$

$(4A + 4B)\cos\theta + (4B - 4A)\sin\theta = 6\sin\theta$

$4A + 4B = 0$

$4B - 4A = 6$

$8B = 6$

$B = 3/4$

$4A + 4(3/4) = 0$

$A = -3/4$

$y_p = -3/4\cos\theta + 3/4\sin\theta$

$\therefore y_s = e^{-2x}(C_1\cos\theta + C_2\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

steady State equation: $y' = 3/4\sin\theta + 3/4\cos\theta = 0$

$$\frac{3/4\sin\theta}{3/4\cos\theta} = \frac{-3/4\cos\theta}{3/4\cos\theta}$$

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where $a = 1$, $b = 4$, $c = 5$

$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 5}}{2 \times 1}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$k = -2 \pm i$

$y_n = e^{-2x}(c_1\cos x + c_2\sin\theta)$

let $y = A\cos\theta + B\sin\theta$

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$-A\cos\theta - B\sin\theta + 4(-A\sin\theta + B\cos\theta) + 5(A\cos\theta + B\sin\theta) = 6\sin\theta$

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steady state equation: $y' = 3/4\sin\theta + 3/4\cos\theta = 0$

$\frac{3/4\sin\theta}{3/4\cos\theta} = \frac{-3/4\cos\theta}{3/4\cos\theta}$

$\tan\theta = 1$

$\theta = \tan^{-1}(-1)$

$\theta = -45^\circ$

2 $EI\frac{d^2y}{dx^2} = \frac{\omega}{2}(L-x)^2$

$EIK^2 = 0$

$K^2 = 0$

$K = \pm\sqrt{0}$

$K = \pm 0$

$y = e^{2x}(A+Bx)$

$y_n = A+Bx$

$y_p = Fx^2 + Gx^3 + Hx^4$

$y' = 2Fx + 3Gx^2 + 4Hx^3$

$y'' = 2F + 6Gx + 12Hx^2$

$6EI(2F + Gx + 12Hx^2) = \frac{\omega}{2}(L-x)^2$

$2FEI + 6GxEI + 12HEIx^2 = \frac{\omega}{2}(L-x)^2$

$4FEI + 12GxEI + 24HEIx^2 = \omega(L^2 - 2Lx + x^2)$

$24HEI = \omega$ comparing both equations

$H = \frac{\omega}{24EI}$

$12GEI = 2\omega L$

$G = \frac{2\omega l}{12EI} = \frac{-\omega}{6EI}$

$4FEI = \omega l^2$

$F = \frac{\omega L^2}{12EI} = -\frac{\omega}{6EI}$

$4FEI = \omega L^2$

$F = \frac{\omega L^2}{4EI}$

$y = \left[\frac{\omega L}{4EI}\right]x^2 - \left[\frac{\omega L}{6EI}\right]x^3 - \left[\frac{\omega}{24EI}\right]x^4$

$= \frac{\omega L^2 x^2}{4EI} - \frac{\omega L x^2}{6EI} + \frac{\omega x^4}{24EI}$

$y = \frac{6\omega L^2 x^2 - 4\omega L x^2 + \omega x^4}{24EI}$

$$y_s = A + Bx + \frac{\omega}{24EI}\left[6L^2x^2 - 4Lx^2 + x^4\right]$$

at $y=0$, $x=0$ and $y'=0$

$$0 = A + B(0) + \frac{\omega}{24EI}\left[6L^2(0)^2 - 4L(0)^2 + (0)^4\right]$$

$$A = 0$$

$$y' = B + \frac{\omega}{24EI}\left[12L^2x - 8Lx + 4x^3\right]$$

$$0 = B + \frac{\omega}{24EI}\left[12L^2(0) - 8L(0) + 4(0)^3\right]$$

$$B = 0$$

$$y = Fx^2 + Gx^3 + Hx^4$$

$$= \frac{\omega L^2x^2}{4EI} - \frac{\omega Lx^2}{6EI} + \frac{\omega x^4}{24EI}$$

$$y = \frac{\omega}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{\omega x^2}{24EI}\left[6L^2x^2 - 4Lx + x^2\right]$$

when $x = l$

$$y = \frac{\omega L^2}{24EI}\left[6L^2 - 4L^2 + L^2\right]$$

$$y = \frac{3\omega L^4}{24EI}$$

$$y = \frac{\omega L^4}{8EI}.$$

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