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Assignment 4

1 $[1-x^2]\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

General Equation

$$W^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!} + \cdots$$

for $[1-x^2]y''$ let $u = y''$, $u' = y'''$, $u'' = y^{iv}$, $u^n = y^{n+2}$

$v = v^0 = 1-x^2$, $v' = -2x$, $v'' = -2$, $v''' = 0$

for xy'

$u = y'$, $u = y''$, $u'' = y'''$, $u^n = y^{n+1}$

$v = v^0 = x$, $v' = 1$, $v'' = 0$

$W_2^n = x \cdot y^{n+1} + n \cdot y^n \cdot 1$

$= xy^{n+1} + ny^n$

for $y = y^n = W_3^n$

$W^n = W_1^n + W_2^n + W_3^n$

$W^n = [1-x^2]y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2(xy^{n+1} + ny^n)$

$0 = (1-x^2)y^{n+2} - 2xny^{n+1} - (n^2-n)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

If $x = 0$

$0 = y^{n+2} - (n^2-n)y^n - 2ny^n + 2y^n$

$y^{n+2} = (n^2 - n + 2n - 2)y^n$

$y^{n+2} = (n^2 + n - 2)y^n$

for $n = 1$

$y^3 = y''' = (1^2 - 1 + 2 - 2)y' = 0$

for $n = 2$

$y^4 = y^{iv} = (2^2 + 2 - 2)y'' = 4y''$

for $n = 3$

$y^5 = y^v = (3^2 + 3 - 2)y''' = 10y''' = 0$

for $n = 4$

$y^6 = y^{vi} = (4^2 + 4 - 2)y^{iv} = 18y^{iv} = 18(4y'')$

for $n = 5$

$y^7 = y^{vii} = (5^2 + 5 - 2)y^v = 28y^v = 0$

for $n = 6$

$y^8 = y^{viii} = (6^2 + 6 - 2)y^{vi} = 40y^{vi} = (40 \times 18 \times 4)y''$

$$y = 1 + f(x) + y'f'(x) + y''\frac{f''(x)}{2!} + \cdots$$

$$y = 1 + y^{\circ} + y' + \frac{y''}{2!} + \frac{4y''}{4!} + \frac{18y''(4) + 40(18\times4)y''}{6!\qquad 8!}$$

$$y = 1 + y^{\circ} + y' + y''\left(\tfrac{1}{2} + \tfrac{1}{6} + \tfrac{1}{10} + \tfrac{1}{14}\right)$$

2(i) $L(3e^{-4t} - 5e^{4t}) = \dfrac{3}{s+4} - \dfrac{5}{s-4}$

(ii) $L[\sin 4t + \cos 4t] = \dfrac{4}{s^2+4^2} + \dfrac{s}{s^2+4^2}$

$$= \frac{4+s}{s^2+16}$$

(iii) $L[t^3 + 2t^2 - t + 4] = \dfrac{3!}{s^{3+1}} + \dfrac{2(2!)}{s^{2+1}} - \dfrac{1!}{s^{1+1}} + \dfrac{4}{s}$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(iv) $L[e^{-2t}\cos 5t] = \dfrac{s+2}{(s+2)^2+5^2} = \dfrac{s+2}{s^2+4s+29}$

(v) $L[t\sin 3t] = (-1)^n \cdot \dfrac{d^n}{dx^n}[f(t)] = -1\dfrac{d}{dx}\left[\dfrac{1\cdot 3}{s^2+3^2}\right]$ $du = 0$ $dv = 2s$

$$= \left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$$

(vi) $L\left[\dfrac{e^{-t} - e^{-2t}}{t}\right] = \dfrac{\left[\frac{1}{s+1} - \frac{1}{s+2}\right]}{s^{\frac{1}{1+1}}} = \left[\dfrac{1}{s+1} - \dfrac{1}{s+2}\right]s^2$

$$= \frac{s^2}{(s+1)(s+2)}$$

(vii) $L[e^{4t}\cos 2t] = \dfrac{s}{(s-4)^2+2^2} = \dfrac{s}{s^2-8s+20}$

(viii) $L[t\sin 2t] = -1 \cdot \frac{d}{dx}\left[\frac{2}{s^2+2^2}\right]$ $du=0$ $dv=2s$

$= -1\left[\frac{-4s}{(s^2+2^2)^2}\right]$ $du=0$ $dv=2s$

$= \frac{4s}{(s^2+4)^2}$

(ix) $t^3+4t^2+5 = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+1}} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(x) $e^{3t}[t^2+4] = t^2e^{3t}+4e^{3t} = \frac{2!}{[s-3]^{1+1}} + \frac{4}{s-3}$

$= \frac{2}{[s-3]^2} + \frac{4}{s-3}$

(xi) $t^2\cos t = (-1)^2 \cdot \frac{d^2}{dx^2}\left[\frac{s}{s^2+1}\right] = \frac{d}{dx}\left[\frac{d}{dx}\left(\frac{s}{s^2+1}\right)\right]$ $du=1$ $dv=2s$

$= \frac{d}{dx}\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right] = \frac{d}{dx}\left[\frac{1-s^2}{[s^2+1]^2}\right]$ $du=-2s$ $dv=4s^3+4s$

$= \left[\frac{-2s^5-4s^3-2s-4s+4s^5}{[(s^2+1)^2]^2}\right] = \left[\frac{2s^5-4s^3-6s}{[s^2+1]^4}\right]$

(xii) $\frac{\sinh 2t}{t} = \frac{1}{2}\ln(s^2-4) - \ln s$

3(i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

at $s=3$

$3-5 = A(3-4)$

$A=2$

at $s=4$

$4-5 = B(4-3)$; $B=-1$

$\therefore L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right] = 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{[s-2][s-4]} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2(2)-6 = A(2-4) \Rightarrow A = 1$

$2(4)-6 = B(4-2) \Rightarrow B = 1$

$L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right] = e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

at $s=0$

$5(0)-8 = A(0-4) \Rightarrow A = 2$

at $s=4$

$5(4)-8 = B(4) \Rightarrow B=3$

$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right] = 2 + 3e^{4t}$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-1)^2 + B(s-3)(s-1) + C(s-3)$

at $s=3$: $3^2-3(3)-4 = A(3-1)^2 \Rightarrow A = -1$

at $s=1$: $1^2-3(1)-4 = C(1-3) \Rightarrow C = 3$

$s^2-3s-4 = [s^2-2s+1]A + (s^2-4s+3)B + (s-3)C$

$= -2A - 4B + C = -3$

$-2(-1) - 4(B) + 3 = -3$

$-4B = -3-3-2 \Rightarrow B = 2$

$L^{-1}\left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right] = -e^{3t} + 2e^{t} + 3te^{t}$

(v) $\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16} = \frac{s-5}{(s+2)^2+4^2}$

$\frac{s-5}{(s+2)^2+4} = (e^{2t}-7)\cos 4t.$