AGBEDE GOODNESS YERIMAH
15/ENG03/1003
CIVIL ENGINEERING.
ENG 881(ENGR MATHS)

(1) $\frac{dy}{dt} + 3y = e^{-2t}$ given that $t=0, y=2$

$L\{dy/dt\} = sY(s) - y(s)$

$L\{3y\} = 3y$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sy(s) - y(s) + 3y(s) = \frac{1}{s+2}$

$3y(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1 + 2(s+2)}{(s+2)}$

$y(s) = \frac{1+2(s+2)}{(s+2)(s+3)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As + 3A + Bs + 2B$

$A + B = 2 \times 3$ — (1)

$3A + 2B = 5 \times 1$ — (2)

$3A + 3B = 6$ — (3)

$3A + 2B = 5$ — (4)

$B = 1$

From eqn (1)

$A + 1 = 2$

$A = 2 - 1 = 1$

$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1}[y(s)] = L^{-1}\left\{\frac{1}{s+2} + \frac{1}{s+3}\right\}$

$y = e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ given that $t=0, y=1$

$L\{3dy/dt\} = 3(sy(s) - y(s))$

$L\{-6y\} = -6y(s)$

$L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$3sy(s) - 3y(s) - 6y(s) = \frac{2}{s^2+2^2}$

$3sy(s) - 6y(s) - 3 = \frac{2}{(s+2)^2}$

$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$

$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$

$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

$2+3s^2+12s+12 = A3s^2 - A12 + 3Bs - 6B + Cs^2 + 4Cs + 4C$

$3A + C = 3$ — (1)

$3B + 4C = 12$ — (2)

$-12A - 6B + 4C = 14$ — (3)

From (1)

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C - 6B + 4C = 14 \times -6$

$-6B + 8C = 28$

$-18B - 24C = -72$

$-18B + 24C = 84$

$48C = -156$

$C = -13/4$

From (2)

$3B = 12 - 4B$

$B = -4/3$

from (1)

$3A = 3 - C$

$3A = 3 - \frac{13}{4}$

$A = -1/12.$

$\frac{2+3(s+2)^2}{(s+2)^3(3s-6)} = \frac{-1/12}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(2s-6)}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1/12}{(s+2)} - \frac{1/3}{(s+2)^3} + \frac{13/4}{3s-6}\right\}$

$y = \frac{-1}{12}e^{-2t} - \frac{1}{3}te^{-4t} + \frac{13}{12}e^{3t}$

$y = \frac{-1}{12}\left[e^{-2t} + 4te^{-4t} + 13e^{3t}\right]$

(3) $\frac{dy}{dt} - 4y = 8$ given that $t=0, y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{-4y\} = -4y(s)$

$L\{8\} = 8/s$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 4y(s) - y(0) = 8/s$

$y(s)(s-4) - 2 = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$8+2s = A(s-4) + Bs$

$8+2s = As - 4A + Bs$

$A+B = 2$

$-4A = 8$

$A = -2$

$B = 2+2 = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}\{y(s)\} = L^{-1}\left[\frac{2}{s} + \frac{4}{s-4}\right]$

$y = -2 + 4e^{4t}$

(4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ given at $t=0$ $y=2, y'=1$

$L\{\frac{d^2y}{dt^2}\} = s^2y(s) - sy(0) - y'(0)$

$L\{-2dy/dt\} = -2sy(s) + 2y(0)$

$L\{5y\} = 5y(s)$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2y(s) - sy(0) - y'(0) - 2sy(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$y(s)(s^2-2s+5) = \frac{1}{s-2} + 2s - 3$

$y(s)(s^2-2s+5) = \frac{2s^2 - 3(s-2)}{(s-2)}$

$y(s) = \frac{1 + 2s^2 - 5s + 6}{(s-2)(s^2-2s+5)}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$

$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs+C}{(s^2-2s+5)}$

$2s^2 - 7s + 7 = A(s^2-2s+5) + (Bs+C)(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$A + B = 2 \rightarrow (1)$

$-2A - 2B + C = -7 \rightarrow (2)$

$5A - 2C = 7 \rightarrow (3)$

From (1)

$B = 2 - A$

From (2)

$-2A - 2(2-A) + C = -7$

$-2A - 4 + 2A + C = -7$

$C = -3$

From (3)

$5A - 2(-3) = 7$

$5A = 7 - 6$

$A = 1/5$

$A + B = 2.$

$\frac{1}{5} + B = 2.$

$B = 2 - 1/5 \quad B = 9/5$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5s-3}{(s^2-2s+5)}$$

$$= \frac{1}{5}\frac{}{(s+2)} + \frac{9/5}{(s^2-2s+5)} \quad \frac{s-3}{(s^2-2s+5)}$$

$$= \frac{1}{5}\frac{}{(s+2)} + \frac{9}{5}\left(\frac{s-1+1}{s^2-2s+5}\right) - \frac{3\times 2/2}{(s^2-2s+5)}$$

$$\frac{1}{5}\frac{}{(s-2)} + \frac{9}{5}\left(\frac{s-1+1}{(s-1)^2+4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}[y(s)] = L^{-1}\left[\frac{1}{5}\frac{}{s-2} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1\times 2}{2(s-1)^2+2^2}\right] - 3/2\left[\frac{2}{(s-1)^2+2^2}\right]\right]$$

$$L^{-1}\left[\frac{1/5}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right] - \frac{3}{2}\left[\frac{2}{(s-1)^2+2^2}\right]\right]$$

$$y = \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t\cos 2t + \frac{1}{2}(e^t\sin 2t)\right] - \frac{3}{2}(e^t\sin 3t) //$$

(5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ given ~~$t=t+4=0$~~

$y=0, y'=2$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y(s) - sy(s) - y'(s)$$

$$L\{-6\,dy/dt\} = -6sy(s) + 6y(s)$$

$$L\{8y\} = 8y(s)$$

$$L\{e^{3t}\} = \frac{1}{s-3}$$

$$s^2y(s) - sy(s) - y'(s) - 6sy(s) + 6y(s) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$$

$$y(s)(s^2-6s-3) = \frac{1}{s-3} + 2$$

$$y(s)(s^2-6s+8) = 1 + 2(s-3)$$

$$\frac{}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{Bs+C}{(s^2-6s+8)}$$

$$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$$

$$2s-5 = As^2-6As+8A+Bs^2-3Bs+Cs-3C$$

$A+B=0$

$-6A-3B+C=2$

$8A-3C=-5$

$B=-A$ from ③

$-6A+3A+C=2$

$-3A+C=2$ -- ④ $\times -3$

$8A-3C=-5$ -- ⑤ $\times 1$

$9A-3C=-6$

$8A-3C=-5$

$A=-1$

$B=1$

from eqn ④

$C=2-3$

$C=2-1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{(s-1)}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{(s-1)}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$s-1 = A(s-4) + B(s-2)$

$s-1 = As-4A+Bs-2B$

$A+B=1$ - ⑪ $\times -4$

$-4A-2B=-1$ ⑫ $\times 1$

$-4A-4B=-4$

$-4A-2B=-1$

$-2B=-3$

$B=3/2$

$$A = \frac{-1}{2}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{(s-2)} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8} = \frac{-1}{(s-3)} + \left(\frac{-1/2}{s+2)} + \frac{3/2}{s-4}\right)$$

$$L^{-1}\{y'(s)\} = L^{-1}\left[\frac{-1}{(s-3)} - \frac{1/2}{(s-2)} + \frac{3/2}{(s-4)}\right]$$

$$y = -e^{3t} - 1/2 e^{2t} + 3/2 e^{4t}.$$

$$y = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right].$$