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Computer Engg

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ENG 381

Assignment 5

1. $\frac{dy}{dt} + 8y = e^{-3t}$

$y(t) + 3y(t) = e^{-2t}$

$L[y'(t)] = 8y(s) - y(s)$

$L[y(t)] = y(s) + 3y(s) = \frac{1}{s+2}$

$(s-3)y(s) - 2 = \frac{1}{s+2}$

$(s-3)y(s) - \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+2}{s+2}$

$y(s) = \frac{2s+2}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2(-2)+2 = A(-2-3) \Rightarrow A = 1/-5$

$2(B)+2 = B(s+2) \Rightarrow B = 11/5$

$y(s) = 1/5(s+2) + 11/5(s-3) = 1/5 e^{-2t} + 11/5 e^{3t}$

ii $3 \frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(t)$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$

$3s + Y(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$Y(s) [3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$

$3s^2+14 = A(s^2+4) + B(3s-6)$

$4A - 6B = 14$

$-6B = 14 - 12$

$B = -1/3$

$$y(s) = \frac{3}{[3][s-2]} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

iii) $\frac{dy}{dt} - 4y = 8$

$$y'(t) - 4y(s) = 8$$

$$y'(t) - 4y(t) = 8$$

$$s y(s) - y(0) - 4y(s) = 8/s$$

$$(s-4)y(s) = \frac{8}{s} + 2 = \frac{8+2s}{s}$$

$$y(s) = \frac{2s+8}{s(s-4)} = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \Rightarrow A = -2$$

$$2(4)+8 = A(4-4) + B(4)$$

$$B = 4$$

$$\mathcal{L}^{-1} \left[\frac{-2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$\mathcal{L}[y''(t)] = s^2 y(s) - s y(0) - y'(0)$$

$$\mathcal{L}[y'(t)] = s y(s) - y(0)$$

$$\mathcal{L}[y(t)] = y(s)$$

$$s^2 y(s) - s y(0) - y'(0) - 2[s y(s) - y(0)] + 5y(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5] y(s) = \frac{1}{s-2} - (2-s)^2 + 1$$

$$= \frac{1}{s-2} + 2s - 4 + 1 = \frac{1}{s-2} + 2s - 3$$

$$= \frac{1 + (2s-3)(3-2)}{(s-2)} = \frac{1 + 2s^2 - 6s - 3s + 6}{s-2}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = -7 + 4 = -3$$

$$= \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$$

$$= 2e^{2t} - \frac{3}{5} t \sin 2t$$

$$v) \frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t}$$

$$s^2 y(s) - s y(0) - y'(0) - 6s y(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] y(s) + (6-3)y(0) - y'(0) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3} = \frac{2s - 5}{s-3}$$

$$y(s) = \frac{2s - 5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s - 5 = A(s-2)(s-4) + B(s-3)(s-4) + [(s-3)(s-2)]$$

$$2s - 5 = A(s^2 - 6s + 8) + B(s^2 - 7s + 12) + (s^2 - 5s + 6)$$

$$2(3) - 5 = A(3-2)(3-4) \Rightarrow A = -1$$

$$2(4) - 5 = (4-3)(4-2) \Rightarrow C = \frac{3}{2}$$

$$-6A - 7B - 5C = 7$$

$$-6[-1] - 7(B) - 5(\frac{3}{2}) = 7$$

$$-7B = 7 + \frac{15}{2} - 6 = \frac{4 + 15 - 12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$\mathcal{L}^{-1} \left[\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= -e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$