

Assignment 5

(i)  $\frac{dy}{dt} + 3y = e^{-2t}$

$y' + 3y = e^{-2t}$

$sY(s) - y(0) + 3y(s) = \frac{1}{s+2}$

at  $t=0, y=2$

$sY(s) - 2 + 3Y(s) = \frac{1}{s+2}$

$Y(s)(s+3) - 2 = \frac{1}{s+2}$

$Y(s)(s+3) = \frac{1}{s+2} + 2$

$Y(s)(s+3) = \frac{1+2(s+2)}{(s+2)}$

$Y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$Y(s) = \frac{2s+5}{(s+2)(s+3)}$

$2s+5 = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = s(-3) + 2B$

$2(-3)+5 = B(-3+2)$

$-6+5 = -B$

$-1 = -B$

$B=1$

$s = -2$

$2(-2)+5 = A(-2+3)$

$-4+5 = A$

$A=1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

(ii)  $3\frac{dy}{dt} - 6y = \sin t$  at  $t=0, y=1$

$3(sY(s) - y(0)) - 6y(s) = \frac{1}{s^2+1}$

$3(sY(s) - 1) - 6y(s) = \frac{1}{s^2+1}$

$3sY(s) - 3 - 6y(s) = \frac{1}{(s^2+1)}$

$3sY(s) - 6y(s) = \frac{1}{(s^2+1)} + 3$

$Y(s)(3s-6) = \frac{1+3(s^2+1)}{(s^2+1)}$

$Y(s) = \frac{1+3s^2+3}{(s^2+1)(3s-6)}$

$Y(s) = \frac{3s^2+4}{(s^2+1)(3s-6)} = \frac{As+B}{(s^2+1)} + \frac{C}{(3s-6)}$

$3s^2+4 = A(s^2+1) + B(s^2+1) + C(3s-6)$

$3s^2+4 = 3As - 6A + Bs^2 + B + 3Cs - 6C$

$B=3$

$3A=0$

$A=0$

$-6A+B=4$

$3s^2+4 = (As+B)(3s-6) + C(s^2+1)$

$3s^2+4 = 3As^2 - 6A + 3Bs - 6B + Cs^2 + C$

$3A+C=3$

$3A+4+6B=3$

$-6A+3B=0$

$3A+6B=-1$

$-6B+C=4$

$3B=-6A$

$C=4+6B$

$B=6A$

$B=2A$

$3A+12A=-1$

Q7

$$y(s) = \frac{-1}{s-3} + \frac{3/2}{s-4} - \frac{1/2}{s-2}$$

f1

$$y(t) = \left[ -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t} \right]$$

f2

V

$$C = 2 - 4$$

$$-2A + B - 2(2 - A) = -7$$

$$-2A + B - 4 + 2A = -7$$

$$B - 4 = -7$$

$$B = -7 + 4$$

$$B = -3$$

$$-2(-3) + 5C = 7$$

$$6 + 5C = 7$$

$$5C = 7 - 6$$

$$5C = 1$$

$$C = 1/5$$

$$A + 1/5 = 2$$

$$A = 2 - 1/5$$

$$A = 9/5$$

$$\frac{9/5 s + 3}{s^2 - 2s + 5} + \frac{1/5}{s - 2}$$

$$\frac{9s}{s^2 - 2s + 5} - \frac{3}{s^2 - 2s + 5} + \frac{1/5}{s - 2}$$

$$\frac{9s}{(s-1)^2 + 4} - \frac{3}{(s-1)^2 + 4} + \frac{1/5}{s-2}$$

$$\frac{9}{5} \left[ \frac{s+1}{(s-1)^2 + 4} \right] - \frac{3 \times \frac{2}{2}}{(s-1)^2 + 4} + \frac{1/5}{s-2}$$

$$\frac{9}{5} \left[ \frac{s-1}{(s-1)^2 + 4} \right] + \frac{1 \times \frac{1}{2}}{(s-1)^2 + 4} - \frac{3 \left[ \frac{2}{2} \right]}{2 \left[ (s-1)^2 + 4 \right]} + \frac{1/5}{s-2}$$

$$\frac{9}{5} \left[ e^t \cos 2t + \frac{1}{2} e^t \sin 2t \right] - \frac{3}{2} e^t \sin 2t + \frac{1}{5} e^{2t}$$

(5.7.11)

$$-s^2 - 2s + 4$$

$$(v) \frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t} \quad t=0, y=0, y'=2$$

$$s^2 y(s) - s y(0) - y'(0) - 6(s y(s) - y(0)) + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - 2 - 6s y(s) + 8y(s) = \frac{1}{s-3}$$

$$y(s) (s^2 - 6s + 8) - 2 = \frac{1}{s-3}$$

$$y(s) (s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$y(s) (s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) (s-4)(s-2) = \frac{1 + 2(s-3)}{(s-3)}$$

$$y(s) = \frac{2s - 6 + 1}{(s-3)(s-4)(s-2)}$$

$$y(s) = \frac{2s - 5}{(s-3)(s-4)(s-2)}$$

$$2s - 5 = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$2s - 5 = A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)$$

$$s = 4$$

$$8 - 5 = B(4-3)(4-2)$$

$$3 = B(2)$$

$$B = 3/2$$

$$s = 2$$

$$4 - 5 = C(2-3)(2-4)$$

$$-1 = C(-1)(-2)$$

$$-1/2 = C$$

$$s = 3$$

$$6 - 5 = A(3-4)(3-2)$$

$$1 = A(-1)(1)$$

$$A = -1$$

$$3(-1/s) + C = 3$$

$$-1/s + C = 3$$

$$C = 3 + 1/s$$

$$C = 16/5$$

$$-(A+3B) = 0$$

$$-(1/5) + 3B = 0$$

$$1/5 + 3B = 0$$

$$3B = -1/5$$

$$B = -1/15$$

$$f(s) = \frac{-1/s - 2/15}{s^2+1} + \frac{16/5}{3s-6}$$

$$= \frac{-1/s}{s^2+1} - \frac{2/15}{s^2+1} + \frac{16/5}{3s-6}$$

$$y(t) = -1/5 \cos t - 2/15 \sin t + 16/5 e^{2t}$$

$$\textcircled{2} \frac{dy}{dt} - 4y = 8 \quad \text{at } t=0, y=2$$

$$s y(s) - y(0) - 4y(s) = 8/s$$

$$s y(s) - 2 - 4y(s) = 8/s$$

$$y(s)(s-4) = 8/s + 2$$

$$y(s)(s-4) = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$8+2s = \frac{A}{s} + \frac{B}{s-4}$$

$$8+2s = A(s-4) + Bs$$

$$8+2s = As - 4A + Bs$$

$$A+B = 2$$

$$-4A = 8$$

$$A = -2$$

$$-2+B = 2$$

$$B = 4$$

$$1/s \cdot \frac{1}{s(s-2)} = \frac{2}{s} + \frac{4}{s-2}$$

$$y(t) = -2 + 4e^{2t}$$

$$\textcircled{2} \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$$t=0, y=2, y'=1$$

$$s^2 y(s) - s y'(0) - y(0) - 2s y(s) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - s y'(0) - y(0) - 2s y(s) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - s(1) - 2 - 2s y(s) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2s y(s) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2s y(s) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2s y(s) + 5y(s) = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s - 1 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s + 1$$

$$y(s)(s^2 - 2s + 5) = \frac{1 + (2s+1)(s-2)}{(s-2)}$$

$$y(s)(s^2 - 2s + 5) = \frac{1 + 2s^2 - 4s - 3s + 2}{(s-2)}$$

$$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 3}{(s-2)}$$

$$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 7}{(s-2)}$$

$$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 7}{(s-2)}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s^2 - 2s + 5)(s-2)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s-2} + \frac{C}{s-2}$$

$$2s^2 - 7s + 7 = (A+B)(s-2) + C(s^2 - 2s + 5)$$

$$2s^2 - 7s + 7 = As^2 - 2As - 2B + C(s^2 - 2s + 5)$$

$$2s^2 - 7s + 7 = As^2 - 2As + Bs - 2B + Cs^2 - 2Cs + 5C$$

$$A + C = 2$$

$$-2A + B - 2C = -7$$

$$C = 2 - A$$

$$-2A + B - 2(2-A) = -7$$

$$-2A + B - 4 + 2A = -7$$

$$B - 4 = -7$$

$$B = -3$$

$$-2(2-A) + 5C = -7$$

$$-4 + 2A + 5C = -7$$

$$2A + 5C = -3$$

$$2A + 5(2-A) = -3$$

$$2A + 10 - 5A = -3$$

$$-3A + 10 = -3$$

$$-3A = -13$$

$$A = 13/3$$

$$C = 2 - 13/3 = -5/3$$

$$A = 13/3$$

$$B = -3$$

$$C = -5/3$$

$$A = 13/3$$

$$B = -3$$

$$C = -5/3$$

$$A = 13/3$$

$$B = -3$$

$$C = -5/3$$

$$A = 13/3$$

$$B = -3$$

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$$C = -5/3$$

$$A = 13/3$$

$$B = -3$$

$$C = -5/3$$

$$A = 13/3$$