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COMPUTER ENGINEERING

1. $\frac{dy}{dt} + 3y = e^{-2t}$ given that $t=0, y=2$

$y' + 3y = e^{-2t}$

$L[f'(t)] = sL[f(t)] - f(0)$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1 + (s+2)(2)}{s+2}$

$y(s)(s+3) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{2s+5}{(s+2)(s+3)}$

$\frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

FOR A

$1+2s+4 = A(s+3) + B(s+2)$

$2s+5 = As+3A+Bs+2B$

$2s+5 = As+Bs+3A+2B$

$A+B=2$

$3A+2B=5$

$A = 2-B$

$3(2-B)+2B = 5$

$6-3B+2B = 5$

$6 - 5 = 3B - 2B$

$1 = B$

$B = 1$

$A + 1 = 2$

$A = 2 - 1$

$A = 1$

$$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y(t) = e^{-2t} + e^{-3t}$$

2 $3\frac{dy}{dt} - 6y = \sin 2t$ at $y=1, t=0$

$3sy(s) - y(0) - 6y(s) = \frac{2}{s^2+4}$

$3(sy(s) - 1) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 6y(s) = \frac{2}{s^2+4} + 3$

$y(s)(3s-6) = \frac{2+3(s^2+4)}{s^2+4}$

$y(s)(3s-6) = \frac{2+3s^2+12}{s^2+4}$

$y(s)(3s-6) = \frac{3s^2+14}{s^2+4}$

$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$

$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{(s^2+4)} + \frac{C}{(3s-6)}$

At C, $3s-6=0$

$3s=6$

$s=6/3$

$s=2$

$(3s-6) \times \frac{3s^2+14}{(s^2+4)(3s-6)}$

at $s=2$

$= \frac{3(2)^2+14}{2^2+14} = \frac{3(2)^2+14}{(2^2+4)(3)}$

$= \frac{12+14}{4+4} = \frac{26}{8}$

$= \frac{13}{8}$ $\frac{16}{8}$ $C = \frac{13}{4}$ //

$3s^2+14 = A+Bs(3s-6) + C(s^2+4)$

$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

$3s^2+14 = 3Bs^2 + Cs^2 + 3As - 6Bs - 6A + 4C$

$3B+C = 3 \quad -(1)$

$3A-6B = 0 \quad -(2)$

$-6A+4C = 14 \quad -(3)$

$3B+C = 3$

$C = 13/4$

$3B + \frac{13}{4} = 3$

$3B = 3 - \frac{13}{4}$

$3B = -\frac{1}{4}$

$B = -1/4 \div 3$

$B = -1/12$

$3A - 6B = 0$

$3A - 6(-1/12) = 0$

$3A + \frac{6}{12} = 0$

$3A = \frac{-6}{12}$

$A = -6/12 \div 3$

$A = -1/6$

$y(s) \quad \frac{-1/6 + -1/12 s}{(s^2+4)} + \frac{13/4}{(3s-6)}$

$= \frac{-1/6}{(s^2+4)} - \frac{\frac{1}{12}s}{s^2+4} + 13/4 (3s-6)$

$= \frac{-1}{6}\left(\frac{1}{s^2+4}\right) - 1/12\left(\frac{s}{s^2+4}\right) + 13/4\left(\frac{1}{3s-6}\right)$

$A = -2$

$-2 + B = 2$

$B = 2 + 2$

$B = 4$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

4 $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

$$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 5y(0) - y'(0) - 2sy(s) - 2y(0) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - s(2) - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$$

$$y(s)(s^2 - 2s + 5) = \frac{1 + (2s-3)(s-2)}{(s-2)}$$

$$y(s)(s^2 - 2s + 5) = \frac{1 + 2s^2 - 4s - 3s + 6}{(s-2)}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s^2 - 2s + 5)(s-2)} = \frac{A + Bs}{(s^2 - 2s + 5)} + \frac{C}{(s-2)}$$

$$2s^2 - 7s + 7 = (A + Bs)(s-2) + C(s^2 - 2s + 5)$$

$$2s^2 + 7s + 7 = As^2 - 2As + Bs^2 - 2A - 2Bs + Cs^2 - 2Cs + 5C$$

$= -\frac{1}{6}\left(\frac{1}{s^2+2^2}\right)\times\frac{2}{2} - \frac{1}{12}\left(\frac{s}{s^2+4}\right)\frac{2}{2} + \frac{13}{4}\left(\frac{1}{3s-6}\right)$

$-\frac{1}{6}\times\frac{1}{2}\left(\frac{2}{s^2+2^2}\right) - \frac{1}{12}\times\frac{1}{2}\left(\frac{s}{s^2+2^2}\right) + \frac{13}{4}\left(\frac{1}{3s-6}\right)$

$-1/12\left(\frac{2}{s^2+2^2}\right) - 1/12\left(\frac{s}{s^2+2^2}\right) + 13/4\left(\frac{1}{3s-6}\right)$

$-1/12\left(\frac{2}{s^2+2^2}\right) - 1/12\left(\frac{s}{s^2+2^2}\right) + 13/4\times 1/3(s-2)$

$-1/12\left(\frac{2}{s^2+2^2}\right) - 1/12\left(\frac{s}{s^2+2^2}\right) + 13/4\times 1/3\,(s-2)$

$-1/12\ \text{cos sin}\ 2t - 1/12\cos 2t + 13/12\, e^{2t}$

$-1/12\,(\sin 2t + \cos 2t) + \frac{B}{12}e^{2t}$

3 $\frac{dy}{dt} - 4y = 8$ at $t=0$, $y=2$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$sy(s) - 4y(s) = \frac{8}{s} - 2$

$sy(s) - 4y(s) = \frac{8-2s}{s}$

$sy(s)\ (s-4) = \frac{8-2s}{s}$

$y(s) = \frac{8-2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8-2s = A(s-4) + Bs$

$8-2s = As - 4A + Bs$

$A+B = 2$

$-4A = 8$

$A = \frac{8}{-4}$

$A = -2$

$-2+B = 2$

$B = 2+2$

$B = 4$

$y(s) = \frac{-2}{s}$

$y(t) = -2+$

$\frac{d^2y}{dt^2} - 2$

$s^2y(s) - sy$

$s^2y(s) - sy$

$s^2y(s) - s$

$s^2y(s) -$

$y(s)(s^2 -$

$y(s)(s^2 -$

$y(s)(s^2 - 2$

$y(s)(s^2 - 2$

$y(s) =$

$y(s) =$

$2s^2 - 7s +$

$2s^2 + 7s +$

$$y(t) = \frac{-3}{2} e^{t}\sin 2t + \frac{9}{5}\left[e^{t}\cos 2t + \frac{1}{2}e^{t}\sin 2t\right] + \frac{1}{5}e^{2t}$$

5 $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ $t=0,\ y=0,\ y'=2$

$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) - 2 = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$

$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$

$y(s)(s-4)(s-2) = \frac{1 + 2(s-3)}{s-3}$

$y(s) = \frac{2s - 6 + 1}{(s-3)(s-4)(s-2)}$

$\frac{2s-5}{} = \frac{A}{(s-3)} + \frac{B}{(s-4)} + \frac{C}{(s-2)}$

$2s - 5 = A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)$

$s = 4$

$8 - 5 = B(4-3)(4-2)$

$3 = B(2)$

$B = 3/2$

$s = 2$

$4 - 5 = C(2-3)(2-4)$

$-1 = C(-1)(-2)$

$-1/2 = C$

$s = 3$

$6 - 5 = A(3-4)(3-2)$

$1 = A(-1)(1)$

$\left.\right] + \frac{4/5}{(s-2)}$

$A = -1$

$$Y(s) = \frac{-1}{s-3} + \frac{3/2}{s-4} - \frac{1/2}{s-2}$$

$$y(t) = -e^{3t} + 3/2\, e^{4t} - 1/2\, e^{2t}.$$