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15/mt150/102
Computer Engineering.

1) $(1-x)^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$(1-x^2)y''$

$v = 1-x^2 \quad v' = -2x \quad v'' = -2$

$u^n = y'' \quad y^n = y^{n+2}$

$u^n v + n u^{n-1} v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!}$

$= y^{n+2}(1-x^2) + ny^{n+2-1} \cdot -2x + \frac{n(n-1)y^{n+2-2} \cdot \cancel{-2}}{\cancel{2}}$

$= y^{n+2}(1-x^2) - 2xny^{n+1} \cdot n(n-1)y^n$

$- 2xy'$

$v = -2x \qquad v' = -2 \quad v'' = 0$

$u = y' \qquad y^n = y^{n+1}$

$= y^{n+1} \cdot -2x + ny^{n+1-1} \cdot -2$

$= -2xy^{n+1} - 2ny^n$

$2y$

$v = 2 \qquad v' = 0$

$u = y \qquad u^n = y^n = 2y^n$

$= y^{n+2}(1-x^2) - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

at $x = 0$

$y^{n+2} - n(n-1)y^n - 2xy^n + 2y^n$

$= y^{n+2} + (-n(n-1) - 2n + 2)y^n$

15/MTH801/012

$y^{n+2} - (n^2 - n + 2n - 2)y^n = 0$

$y^{n+2} - (n^2 + n - 2)y^n = 0$

$\left[y^{n+2}\right]_0 = (n^2 + n - 2)y^n$

Maclaurin's principle

$y_0 + xy'_0 + \frac{x^2}{2!}\left[y''\right]_0 + \frac{x^3}{3!}\left[y'''_0\right]$

at $n = 1, 2, 3, 4, 5$

$y^{n+2} = (n^2 + n - 2)y^n$

$n = 1$

$y^3 = (1^2 + 1 - 2)y^1 = 0$

$n = 2$

$y^4 = (2 + 2 - 2]y^n$

$= 4y'' = 4(-2)^0 y = -8y^0$

$n = 3$

$y^5 = (3^2 + 2 - 2)\, y'''$

$= 10y''' = 10 \times 0 = 0$

$n = 4$

$y^6 = [4^2 + 4 - 2]y^4$

$= 18y^4 = 18 \times 4y'' = 72(-2y^0) = -144y^0$

$n = 5$

$y^7 = [5^2 + 5 - 2]y^5$

$= 28y^5 = 28 \times 0 = 0$

$= y_0 + xy'_0 + \frac{x^2}{2}\left[xy^0\right] + \frac{x^3}{3!}\left[0\right] + \frac{x^4}{4!}\left[-8y^0\right] + \frac{x^5}{5!}\left[0\right]$

15/mth/501/012

$= y_0 + xy'_0 - x^2y_0 \; \frac{x^4}{3} y_0$

$= \left[1 - x^2 - \frac{x^4}{3}\right] y_0 + xy'_0$

2) $3e^{-4t} - 5e^{4t}$

$L[3e^{-4t}] - L[5e^{4t}]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

$\frac{3(s-4) - 5(s+4)}{(s+4)(s-4)} = \frac{3s - 12 - 5s - 20}{(s+4)(s-4)} = \frac{-2s - 32}{(s+4)(s-4)}$

3) $\sin 4t + \cos 4t$

$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$\frac{4+s}{s^2+4^2}$

$t^3 + 2t^2 - t + 4$

$L(t)^3 = \frac{3!}{s^4} = \frac{6}{s^4}$

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IT/MTH/19/012

$2L[t^3] = \frac{2!}{s^3} = \frac{2}{s^3}$

$L[t] = \frac{1!}{s^2} = \frac{1}{s^2}$

$L[4] = \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$e^{-2t} \cos 5t$

$= \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{(s+2)^2+25} = \frac{\cancel{s+2}}{s^2+4s+4+25} = \frac{s+2}{s^2+4s+29}$

$t \sin 3t$

$(-1)^1 \frac{d}{ds}\left[\frac{s}{s^2+3^2}\right]$

$= -\left[\frac{6s}{s^2+9^2}\right] = \frac{-6s}{(s^2+9)^2}$

$\frac{e^{-t} - e^{-2t}}{t}$

$\frac{e^{-(0)} - e^{-2(0)}}{0} = \frac{1-1}{0} = \frac{0}{0}$

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using i'l

$$\frac{-e^{-t}+2e^{-2t}}{1} = \frac{-e^{-(0)}+2e^{-2(0)}}{1} = -1+2 = 1$$

$$L\left[e^{-t} - e^{-2t}\right]$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$\int_{s=\sigma}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$= \int_{s}^{\infty} \ln \sigma+1 - \ln \sigma+2$$

$$\ln\left(\frac{\sigma+1}{\sigma+2}\right)_{\sigma}^{\infty}$$

$$\ln \frac{\infty+1}{\infty+2} - \frac{s+1}{s+2}$$

$$= -\ln\left(\frac{s+1}{s+2}\right)$$

$e^{4t}\cos 2t$

$$= \frac{s-4}{(s-4)^2+2^2} = \frac{s-4}{(s-4)^2+4} = \frac{s-2}{s^2-8s+16+4} = \frac{s-2}{s^2-8s+20}$$

$t\sin 2t$

$$(-1)^1 \frac{d}{ds}\left[\frac{2}{s^2+4}\right] = \frac{4s}{(s^2+4)^2} = \frac{4s}{s^4+8s^2+16}$$

15/~~mths~~ 01/2012

$t^3 + 4t^2 + 5$

$= L[t^3] + L[4t^2] + L[5]$

$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + \frac{5}{8}$

$= \frac{6}{s^4} + \frac{8}{s^2} + \frac{5}{s^3} = \frac{6+8s+5s^3}{s^4}$

$e^{3t}(t^2+4)$

$= fe^{3t} + fe^{3t}$

$L[t^2 e^{3t}]$

$=(-t)^2 \frac{d^2}{ds^2}\left[\frac{1}{s-3}\right]$

$= \frac{d}{ds}\left[\frac{1}{(s-3)^2}\right] = \frac{d}{ds}\left[\frac{1}{s^2-6s+9}\right]$

$t^2 \cos t$

$(-1)^2 \frac{d}{ds^2}\left[\frac{s}{s^2+1^2}\right]$

$\frac{d}{ds^2}\left[\frac{s}{s^2+1^2}\right] = \frac{d}{ds}\left[\frac{s^3 - 2s^2+1}{(s^2+1)^2}\right]$

$\frac{\sin 2t}{t}$

$\lim \to 0$

15/MATH5016/0,12

$$= \frac{\sinh 2(0)}{0} = 0$$

using l hopital rule

$$\frac{2\cosh 2t}{1} = 2\cosh 2(0) = 2$$

$$\int_{\theta=2}^{\infty} \frac{2}{s^2-4}\, d\theta$$

$$\int_{\theta=2}^{\infty} 2/_{4+s}\, d\theta$$

$$-2\int_0^{\infty} \frac{1}{4+s^2}\, d\theta$$

$$-2/4 \left[\tan^{-1}\theta\right]_{\theta=s}^{\infty}$$

$$-1/2 \tan^{-1}\theta$$

$$1/2 \tan^{-1} s$$

$$(\tan^{-1} s/a)^{-1} = \tan^{-1} 2/s$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-5 = A(s-4) + B(s-3)$$

$$s-5 = As - 4A + Bs - 3B$$

$$B = 1$$

let $s = 3$

$$3-5 = A(3-4) + B(3-3)$$

$$-2 = -A$$

$$A = 2$$

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15/mth501/012

$$\frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2e^{3t} - e^{4t}$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$2s-6 = A(s-4) + B(s-2)$$

$$2s-6 = \cancel{2}A(s-4) + B(s-2)$$

$$s = 4$$

$$2(4) - 6 = A(4-4) + B(4-2)$$

$$2 = 2B$$

$$B = 1$$

$$s = 2$$

$$2(2) - 6 = A(2-4) + B(2-2)$$

$$-2 = 2A$$

$$A = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right]$$

$$= e^{2t} + e^{4t}$$

15/MTH501/003

$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$5s-8 = A(s-4) + Bs$

$5s-8 = As - 4A + Bs$

at $s=4$

$5(4) - 8 = A(4-4) + 4B$

$20 - 8 = 4B$

$4B = 12$

$B = 3$

at $s=0$

$5(0) - 8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$$= \frac{2}{s} + \frac{3}{s-4}$$

$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$$

$= 2 + 3e^{4t}$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)}{(s-3)(s-1)^2}$$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

15/MTH 01/002

at $s=1$

$1^2 - 3(1) - 4 = A(1-1)^2 + B(1-3)(1-1) + C(1-3)$

$-6 = -2C$

$C = 3$

at $s=3$

$3^2 - 3(3) - 4 = A(3-1)^2 + B(3-3)(3-1) + C(3-3)$

$-4 = 4A$

$A = -1$

at $s=0$

$0^2 - 3(0) - 4 = A(0-1)^2 + B(0-3)(0-1) + C(0-3)$

$-4 = A + 3B - 3C$

$-4 = -1 + 3B - 3\times 3$

$-4 = -1 + 3B - 9$

$-4 = -10 + 3B$

$3B = 6$

$B = 2$

$\frac{-1}{\cancel{s-3}} \quad \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

$= e^{3t} + 2e^{t} + 3te^{t}$

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15/MTH201/092

$$\frac{s-5}{s^2+4s+20} = \frac{\cancel{As}\ Bs}{s^2+4s+20}$$

$s-5 = As+B$

let $s=0$

$0-5 = A(0)+B$

$B = -5$

let $s=1$

$s-5 = As+B$

$1-5 = A(1)+B$

$-4 = A+B$

$-4 = A-5$

$A = 5-4$

$A = 1$

$$\frac{s-5}{s^2+4s+20}$$

$s^2+4s+20=0$

$s^2+4s=-20$

$s^2+4s+(4/2)^2 = -20+(4/2)^2$

$s^2+4s+2^2 = -20+4$

$(s+2)^2 = -16$