3) Using the Leibnitz theorem given that

i) $y = x^3 e^{4x}$ determine y^5

$V = x^3$ $\quad$ $U = e^{4x}$

$V' = 3x^2$ $\quad$ $U' = 4e^{4x}$

$V'' = 6x$ $\quad$ $U'' = 4 \cdot 4 e^{4x}$

$V''' = 6$ $\quad$ $U''' = 4 \cdot 4 \cdot 4 e^{4x}$

$V^{iv} = 0$ $\quad$ $U^{n} = 4^{n} e^{4x}$

$$y^n = 4^n e^{4x} x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^3 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x$$
$$+ \frac{n(n-1)(n-2)}{6} 4^{n-3} e^{4x} \cdot 6$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} e^{4x} 3x$$
$$+ n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$y^5 = 4^5 e^{4x} x^3 + 5 \cdot 4 e^{4x} 3x^2 + 5 \cdot 4 \cdot 4^3 e^{4x} 3x$$
$$+ 5 \cdot 4 \cdot 3 \cdot 4^2 e^{4x}$$

iii) $x^2 y'' + x y' + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$

Let $w = x^2 y''$

$V = x^2$ $\quad$ $U = y''$

$V' = 2x$ $\quad$ $U' = y'''$

$V'' = 2$ $\quad$ $U'' = y^{(iv)}$

$V''' = 0$ $\quad$ $U^n = y^{n+2}$

$$w^n = y^{n+2} x^2 + n \cdot y^{n+1} \cdot 2x + \frac{n(n-1)}{\cancel{2}} \cdot y^n \cdot \cancel{2}$$

$$w^n = x^2 y^{n+2} + n 2x y^{n+1} + n(n-1) y^n$$

Let $p = xy'$

$V = x$ $\quad$ $U = y'$

$V' = 1$ $\quad$ $U' = y''$

$V'' = 0$ $\quad$ $U^n = y^{n+1}$

$$P^n = y^{n+1} \cdot x + n \cdot y^n \cdot 1$$

$$P^n = xy^{n+1} + ny^n$$

$$S = y$$

$$S^n = y^n$$

$$y^{n+2} \cdot x^2 + n \cdot 2xy^{n+1} + n(n-1)y^n + y^{n+1}x + ny^n + y^n = 0$$

$$y^{n+2} x^2 + (n \cdot 2xy^{n+1} + xy^{n+1}) + [n(n-1)y^n + ny^n + y^n] = 0$$

$$x^2 y^{n+2} + (2n+1)x y^{n+1} + [n(n-1) + 1 + n] y^n = 0$$

$$x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$$

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Assignment 3

~~Answer~~

① $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$y'' = 2e^{x^2+x} + (2x+1)^2 e^{x^2+x}$

$y'' = y'(2x+1) + 2y$

$y' = (2x+1)e^{x^2+x}$

$y'(2x+1) = (2x+1)e^{x^2+x}(2x+1)$

$= (2x+1)^2 e^{x^2+x}$

$2y = 2 \cdot e^{x^2+x}$

$\therefore y'(2x+1) + 2y = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$\therefore y'' = y'(2x+1) + 2y$

$\therefore 2e^{x^2+x} + (2x+1)^2 e^{x^2+x} = (2x+1)^2 e^{x^2+x} + 2e^{x^2+x}$

$N = y''$

$N^n = y^{n+2}$

$P = y'(2x+1)$

$V = 2x+1$

$V' = 2 \qquad u = y'$

$V'' = 0 \qquad u' = y^{n+1}$

$P^n = y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2$

$S = 2y$

$S^n = 2y^n$

$N^n = P^n + S^n$

$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$