ASSIGNMENT V

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(1) $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0, y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{3y\} = 3y(s)$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2(s+2)}{(s+2)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As + 3A + Bs + 2B$

$A+B = 2 \quad \times 3$

$3A + 2B = 5 \quad \times 1$

$3A + 3B = 6$ —— (3)

$3A + 2B = 5$ —— (4)

(3) − (4)

$B = 1$

from eqn (1)

$A + 1 = 2$

$A = 2 - 1 = 1$

$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{s+2} + \frac{1}{s+3}\right\}$

$y = e^{-2t} + e^{-3t}$

2) $\frac{3dy}{dt} - 6y = \sin 2t$ given that at $t=0, y=1$

$L\{3\,dy/dt\} = 3(sy(s) - y(0))$

$L\{-6y\} = -6y(s)$

$L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$

$3sy(s) - 6y(s) - 3 = \frac{2}{(s+2)^2}$

$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$

$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$

$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

$2+3s^2+12s+12 = A3s^2 - A12 + 3Bs - 6B + Cs^2 + 4Cs + 4C$

$3A + C = 3$ —— (1)

$3B + 4C = 12$ —— (2)

$-12A - 6B + 4C = 14$ —— (3)

from (1).

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C - 6B + 4C = 14 \quad \times -6$

$\times 3$

$-6B + 8C = 28$

$-18B - 24C = -72$

$-18B + 24C = 84$

$48C = -156$

$C = 13/4$

from (2)

$3B = 12 - 13$

$B = \frac{-1}{3}$

from (1)

$3A = 3 - C$

$3A = 3 - \frac{13}{4}$

$A = -\frac{1}{12}$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-1/12}{(s+2)} + \frac{1/3}{(s+2)^2} + \frac{13/4}{(3s-6)}$$

$$L^{-1}\{y_{(s)}\} = L^{-1}\left\{\frac{-1/12}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(3s-6)}\right\}$$

$$y = \frac{-1}{12}e^{-2t} - \frac{1}{3}te^{-4t} + \frac{13}{12}e^{2t}.$$

$$y = \frac{-1}{12}\left(e^{-2t} + 4te^{-4t} - 13e^{2t}\right)$$

3. $\frac{dy}{dt} - 4y = 8$ given that $t=0$, $y=2$

$L\{dy/dt\} = sy_{(s)} - y_{(0)}$

$L\{-4y\} = -4y_{(s)}$

$L\{8\} = 8/s$

$sy_{(s)} - y_{(0)} - 4y_{(s)} = 8/s$

$sy_{(s)} - 4y_{(s)} - y_{(0)} = 8/s$

$y_{(s)}(s-4) + 2 = \frac{8}{s} + 2.$

$y_{(s)}(s-4) = \frac{8+2s}{s}$

$y_{(s)} = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$8+2s = A(s-4) + Bs$

$8+2s = As - 4A + Bs$

$A+B = 2$

$-4A = 8$

$A = -2$

$B = 2+2$

$B = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}\{y_{(s)}\} = L^{-1}\left\{\frac{2}{s} + \frac{4}{s-4}\right\}$

$y = -2 + 4e^{4t}.$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ ~~given that at t=0~~,

$y = 2,\ y' = 1$

$L\{d^2y/dt^2\} = s^2y_{(s)} - sy_{(0)} - y'_{(0)}$

$L\{-2\,dy/dt\} = -2sy_{(s)} + 2y_{(0)}$

$L\{5y\} = 5y_{(s)}$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2y_{(s)} - sy_{(0)} - y'_{(0)} - 2sy_{(s)} + 2y_{(0)} + 5y_{(s)} = \frac{1}{s-2}$

$s^2y_{(s)} - 2sy_{(s)} + 5y_{(s)} - 2s - 1 + 4 = \frac{1}{s-2}$

$y_{(s)}(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$y_{(s)}(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-2)}$

~~$y_{(s)}(s^2-2s+5$~~

$y_{(s)} = \frac{1 + 2s^2 - 7s + 6}{(s-2)(s^2-2s+5)}$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{(s-2)} + \frac{Bs + C}{(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$A + B = 2$ — ①

$-2A - 2B + C = -7$ — ②.

$5A - 2C = 7$ — ③

from ①

$B = 2 - A$.

from ②

$-2A - 2(2 - A) + C = -7$

$\cancel{-2A} - 4 + \cancel{2A} + C = -7$

$C = -3$

from ③.

$5A - 2(-3) = 7$

$5A = 7 - 6$

$A = 1/5$

$A + B = 2$

$\frac{1}{5} + B = 2$

$B = 2 - 1/5$

$B = 9/5$

$$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{1/5}{(s-2)} + \frac{\frac{9}{5}s - 3}{(s^2 - 2s + 5)}$$

$$= \frac{\frac{1}{5}}{(s+2)} + \frac{9/5\, s}{(s^2 - 2s + 5)} - \frac{3}{(s^2 - 2s + 5)}$$

$$= \frac{\frac{1}{5}}{(s+2)} + 9/5\left(\frac{s - 1 + 1}{s^2 - 2s + 5}\right) - \frac{3 \times 2/2}{(s^2 - 2s + 5)}$$

$$\frac{\frac{1}{5}}{(s+2)} + \frac{9}{5}\left(\frac{s - 1 + 1}{(s-1)^2 + 4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2 + 4}\right)$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{\frac{1}{5}}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2 + 2^2} + \frac{1 \times \frac{2}{2}}{(s-1)^2 + 2^2}\right] - 3/2\left(\frac{2}{(s-1)^2 + 2^2}\right)\right\}$$

$$= L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2 + 2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2 + 2^2}\right)\right] - \frac{3}{2}\left(\frac{2}{(s-1)^2 + 2^2}\right)\right\}$$

$$y = \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^{t}\cos 2t + \frac{1}{2}(e^{t}\sin 2t\right] - 3/2\,(e^{t}\sin 2t).$$

5) $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$ given that at $t = 0$

$y = 0,\ y' = 2$.

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2 y(s) - s y(0) - y'(0)$.

$L\left\{-6\,dy/dt\right\} = -6sy(s) + 6y(0)$.

$L\{8y\} = 8y(s)$.

$L\{e^{3t}\} = \frac{1}{s-3}$

$$s^2 y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$$

$$y(s)\,(s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$y(s)\,(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s - 5}{(s-3)(s^2 - 6s + 8)}$$

$$\frac{2s - 5}{(s-3)(s^2 - 6s + 8)} = \frac{A}{(s-3)} + \frac{Bs + C}{(s^2 - 6s + 8)}$$

$$2s - 5 = A(s^2 - 6s + 8) + (Bs + C)(s-3)$$

$$2s - 5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$$

$A + B = 0$

$-6A - 3B + C = 2$

$8A - 3C = -5$

$B = -A$ from ①

$-6A + 3A + C = 2$

$-3A + C = 2$ —— ④ $\times -3$

$8A - 3C = -5$ —— ⑤ $\times 1$

$\cancel{4 - ⑥}$

$9A - 3C = -6$

$8A - 3C = -5$

$A = -1$

$B = 1$

from ④

$C = 2 - 3$

$C = -1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$s - 1 = A(s-4) + B(s-2)$

$s - 1 = As - 4A + Bs - 2B$

$A + B = 1$ —— ① $\times -4$

$-4A - 2B = -1$ —— ② $\times 1$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = \frac{3}{2}$

$A = -\frac{1}{2}$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \left(\frac{-1/2}{(s-2)} + \frac{3/2}{(s-4)}\right)$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1}{(s-3)} - \frac{1/2}{(s-2)} + \frac{3/2}{(s-4)}\right\}$$

$$y = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$

$$y = \frac{1}{2}\left(2e^{3t} + e^{2t} - 3e^{4t}\right)$$