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15/ENG01/020
Chemical Engineering.

i) $dy + 3y = e^{-2t}$

$t=0 \quad y=2$

$s\,y(s) + y(s) + 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) + 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$L^{-1}\left\{\frac{2s+5}{(s+2)(s+3)}\right\} = y(t)$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3)+B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

when $s = -2$

$-4+5 = A(1) + 0$

$A = 1$

when $s = -3$

$-6+5 = A(0) + B(-1)$

$-B = -1$

$B = 1$

$y(t) = L^{-1}\left[\frac{1}{s+2} - \frac{1}{s+3}\right]$

$y(t)\; e^{-2t} + e^{-3t}$

ii) $3\frac{dy}{dt} - 6y = \sin 2t$

at $t=0 \quad y=1$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3y(s)(s-2) - 3 = \frac{2}{s^2+4}$

$3y(s)(s-2) = \frac{2}{s^2+4} + 3$

$\frac{2 + 3s^2 + 12}{s^2+4}$

$y(s) = \frac{3s^2+14}{3(s-2)(s^2+4)} = \frac{3s^2+14}{(3s-6)(s^2+4)}$

$y(t) = L^{-1}\left[\frac{3s^2+14}{(3s-6)(s^2+4)}\right]$

$\frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}$

$3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$

at $s = 2$

$12+14 = A(8) + (Bs+C)(0)$

$26 = 8A$

$A = 13/4$

Using the method of coefficients

$3 = A+B \; --- 1$

$3 = 13/4 + 3B$

$3B = 3 - 13/4$

$\frac{12-13}{4}$

$3B = -1/4 \qquad B = -1/12$

$14 = 4A - 6C$

$14 = 4 \times \frac{13}{4} - 6C$

$14 = 13 - 6C$

$1 = -6C$

$C = -1/6$

$y(t) = L^{-1}\left[\frac{13}{14} \times \frac{1}{3s-6} + \frac{-\frac{1}{12}s - \frac{1}{6}}{s^2+4}\right]$

$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2} - \frac{1}{12}s \times \frac{1}{s^2+4} - \frac{1}{6} \times \frac{1}{s^2+4}\right]$

$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2}\right] + L^{-1}\left[-\frac{1}{12} \times \frac{s}{s^2+4}\right]$

$+ L^{-1}\left(\frac{1}{6\times 2} + \frac{2}{s^2+4}\right)$

$= \frac{13}{12} e^{2t} - \frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t$

3) $\frac{dy}{dt} - 4y = 8$ at $t=0$, $y=2$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y(s)(s-4) = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$y(s) = L^{-1}\frac{8+2s}{s(s-4)} = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$\frac{8+2s}{s(s-4)} = \frac{A(s-4)+B(s)}{s(s-4)}$

$8+2s = A(s-4) + B(s)$

When $s=4$

$8+8 = A(0) + 4B$

$B = 4$

When $s=0$

$8 = -4A + 0$

$-4A = 8$

$A = -2$.

$y(s) = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$

$y(s) = -2 + 4e^{4t}$

IV) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(s)) + 5y(s) = \frac{1}{s-2}$

at $t=0$ $y=2$ $y'=1$

$s^2y(s) - s(2) - 1 - 2(sy(s) - 2) + 5y(s)$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) =$

$s^2y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) - 2s + 3 = 1/s-2$

$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + \frac{2s}{1} + \frac{3}{1}$

$\frac{1 + 2s^2 - 4 + 3s - 6}{s-2}$

$y(s)\ s^2 - 2s + 5) = \frac{2s^2 + 3s - 9}{s-2}$

$y(s) = \frac{2s^2 + 3s - 9}{(s-2)(s^2 - 2s + 5)}$

$L^{-1}\frac{2s^2 + 3s - 9}{(s-2)(s^2 - 2s + 5)} = \left[\frac{A}{(s-2)} + \frac{Bs + C}{s^2 - 2s + 5}\right]$

$2s^2 + 3s - 9 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

at $s = 2$

$8 + 6 - 9 = A(4 - 8 + 5) + 0$

$A = 5$

Using the method of Coefficient

$2 = A + B$

$2 = 5 + B$

$B = -3$

$3 = -2A - 2B + C$

$3 = -2(5) - 2(-3) + C$

$3 = -10 + 6 + C$

$3 = -4 + C$

$C = -7$

$$y(s) = \left[\frac{5}{s-2} + \frac{(-3s-7)}{s^2-2s+5}\right]$$

$$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[\frac{3s}{(s-1)^2+4}\right] -$$

$$L^{-1}\left[\frac{7}{(s-1)^2+4}\right]$$

$$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[\frac{s+5}{(s-1)^2+4}\right] - L^{-1}\left[\frac{7}{2}\times\frac{2}{(s-1)^2+4}\right]$$

$$= 5e^{2t} - 3e^{t}\cos 2t - 7/2\, e^{t}\sin 2t$$

V) $\frac{\partial^2 y}{\partial t^2} - 6\frac{\partial y}{\partial t} + 8y = e^{3t}$

at $t=0, y=0, y'=2$

$\Rightarrow s^2 y(s) - s y(0) - y'(0) - 6(s y(s) - y(0)) + 8y(s)$

$\Rightarrow s^2 y(s) - 0 - 2 - 6s y(s) + 6(0) + 8y(s)$

$= \frac{1}{s-3}$

$\Rightarrow s^2 y(s) - 2 - 6s y(s) + 8y(s) = \frac{1}{s-3}$

$\Rightarrow y(s)(s^2-6s+8) = \frac{1}{s-3} + \frac{2}{1}$

$y(s)(s^2-6s+8) = \frac{1+2s-6}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$

$y(s) = L^{-1}\left[\frac{2s-5}{(s-3)(s-4)(s-2)}\right]$

$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$

$\frac{2s-5}{(s-3)(s-4)(s-2)} \Rightarrow A(s-4)(s-2) + B(s-3)(s-2) + \frac{C(s-3)(s-4)}{(s-3)(s-2)(s-4)}$

$2s-5 = A(s-4)(s-4) + B(s-3)(s-2) + C(s-3)(s-4)$

at $s=2$

$4-5 = A(0) + B(0) + C(-1)(-2)$

$-1 = 2C$

$C = -1/2$

at $s=3$

$6-5 = A(-1)(1) + B(0) + C(0)$

$1 = -A$

$A = -1$

at $s=4$

$8-5 = A(0) + B(2)(1) + C(0)$

$3 = 2B$

$B = 3/2$

$y(s) = L^{-1}\left[\frac{-1}{s-3} + \frac{3}{2}\times\frac{1}{s-4} - \frac{1}{2}\times\frac{1}{s-2}\right]$

$y(s) = -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$