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AF/ENG/04/033
ELECT | ELECT ENGR
ENG 381
ASSIGNMENT 5

1) $\frac{dy}{dt} + 3y = e^{-2t}$ $t = 0$ $y = 2$

$y(0) = 2$

$y^{(1)} + 3y^{(0)} = e^{-2t}$

$Sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$Sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] = \frac{1}{s+2} + \frac{2}{1} = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A = \left.\frac{1+2s+4}{s+3}\right|_{s=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

$B = \left.\frac{1+2s+4}{s+3}\right|_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$ $y(0) = 1$

$3y^{(1)} - 6y = \sin 2t$

$3(Sy(s) - y(0)) - 6y(s) = 2/s+4$

$3sy(s) - 3y(0) - 6y(s) = 2/s+4$

$y(s)[3s-6] = \frac{2}{s+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$

$y(s)[3s-6] = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+14)}$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{(3s-6)}$$

$$C: \left.\frac{3s^2+14}{s^2+4}\right|_{s=2} = \frac{3(2)^2+14}{2^2+4} = \frac{13}{4}$$

$$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$$

Comparing coefficient

$$3 = 3B + C$$

$$3 = 3B + \frac{13}{4} \quad \left(\text{where } C = \frac{13}{4}\right)$$

$$3A - 6B = 0$$

$$3A = 6B$$

$$3A = 6 \times \frac{-1}{12} \quad A = -\frac{1}{6}$$

$$y(s) = \frac{-\frac{1}{6} - \frac{1}{12}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$\frac{-\frac{1}{6}}{s^2+4} - \frac{\frac{1}{12}s}{s^2+4} + \frac{\frac{13}{4}}{3s-6}$$

$$= -\frac{1}{6} \cdot \frac{1}{s^2+4^2} - \frac{1}{12}\frac{s}{s^2+2^2} + \frac{13}{4} \cdot \frac{1}{3(s-2)}$$

$$= -\frac{1}{6} \cdot \frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + \frac{13}{12}\left[\frac{1}{s-2}\right]$$

$$y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$$

$$y(t) = \frac{1}{12}\left[13e^{2t} - \cos 2t - \sin 2t\right]$$

3) $\frac{dy}{dt} - 4y = 8$ given that $t=0$ $y=2$ $y(0)=2$

$$y^{(1)} - 4y = 8$$

$$Sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$Sy(s) - 2 - 4y(s) = \frac{8}{s}$$

$7 - 1 = -2C$

$C = -3$

$$y(s) = \frac{\frac{1}{5}}{s-2} + \frac{\frac{9s-3}{5}}{s^2-2s+5} - \frac{1}{5} \cdot \frac{1}{s-2} + \frac{\frac{9s}{3}}{(s+1)^2+4} - \frac{3}{(s+1)^2}$$

$$y(s) = \frac{1}{5} \cdot \frac{1}{s-2} + \frac{9}{5} \frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$y(s) = \frac{1}{5} \cdot \frac{1}{s-2} + \frac{9}{5} \frac{(s+1)}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$y(s) = \frac{1}{5} \cdot \frac{1}{s-2} + \frac{9}{5} \frac{s+1}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2} \cdot \frac{2}{2}$$

$$y(t) = \frac{1}{5} e^{2t} + \frac{9}{5} e^{-t} \cos 2t - 2e^{-t} \sin 2t$$

$$y(t) = \frac{1}{5} \left[e^{2t} + \frac{9}{\cancel{5}} e^{-t} \cos 2t - 10 e^{-t} \sin 2t \right]$$

$$= \frac{1}{5} \left[e^{2t} + e^{-t} (9 \cos 2t - 10 \sin 2t) \right]$$

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t} \qquad \text{at } t = 0 \quad y = 0 \quad y^{(1)} = 2$$

$$y^{(1)} - 6y^{(0)} + 8y = e^{3t}$$

$$s^2 y(s) - s y(0) - y'(0) - 6[s y(s) - y(0)] + 8 y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - 2 - 6s y(s) + 8 y(s) = \frac{1}{s-3}$$

$$y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + \frac{2}{1} = \frac{1+2(s-3)}{s-3} = \frac{1+2s-6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} - \frac{C}{s-4}$$

$$A: \left. \frac{2s-5}{(s-2)(s-4)} \right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$$

$$B: \left. \frac{2s-5}{(s-2)(s-4)} \right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = -\frac{1}{2}$$

$$C: \left. \frac{2s-5}{(s-3)(s-2)} \right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$$

$$y(s) = \frac{-1}{s-3} - \frac{1}{2} \cdot \frac{1}{s-2} + \frac{3}{2} \cdot \frac{1}{s-4}$$

$$y(t) = -e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$

$$y(s)[s-4] = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$A: \left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$$

$$B: \left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$$

$$y(s) = \frac{-2}{s} + \frac{4}{s-4}$$

$$y(t) = -2 + 4e^{4t}$$

7) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2t}$ at $t=0$, $y=2$, $y^{(1)}=1$

$y(s) = 2$ $y(0) = 1$

$$y^{(2)} - 2y^{(1)} + 5y = e^{2t}$$

$$(s^2y(s) - sy(s) - y'(0)) - 2(sy(s) - y(s)) + 5y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$y(s)[s^2 - 2s + 5] = \frac{1}{s+2} + \frac{2s}{1} - \frac{3}{1} = \frac{1 + 2s(s-2) - 3(s-2)}{s-2}$$

$$y(s)[s^2 - 2s + 5] = \frac{1 + 2s^2 - 4s - 3s + 6}{s-2} = \frac{2s^2 - 7s + 7}{s-2}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + 6}{s^2 - 2s + 5}$$

$$A: \left.\frac{2s^2 - 7s + 7}{s^2 - 2s - 5}\right|_{s=-2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$2 = A + B$$

$$2 = \frac{1}{5} + B \qquad B = \frac{9}{5}$$

$$7 = 5A - 2C$$

$$7 = 5\left(\frac{1}{5}\right) - 2C$$