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NF/ENG/06/033

400 LEVEL

ELECT/ELECT ENGR

ENG 381

ANSWER

(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Convert equation into an homogeneous equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$m^2 - m - 2 = 0$$

$$(m^2 + m)(-2m - 2) = 0$$

$$m(m+1) - 2(m+1) = 0$$

$$(m+1)(m-2) = 0$$

$$m = -1, \quad m = 2$$

$$y = Ae^{-x} + Be^{2x}$$

$$y = C$$

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - 0 - 2C = 8$$

$$-2C = 8$$

$$C = \frac{8}{-2} = -4$$

$$C = -4$$

G.S = we have $Ae^{-x} + Be^{2x} - 4$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{3x}$

$\frac{d^2y}{dx^2} - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \sqrt{4}$

$m = \pm 2$

$y = C\cosh 2x + D\sinh 2x$ C.F

$y = Ce^{3x}$

$\frac{dy}{dx} = 3Ce^{3x}$

$\frac{d^2y}{dx^2} = 9Ce^{3x}$

$9Ce^{3x} - 4Ce^{3x} = 10e^{3x}$

$5Ce^{3x} = 10e^{3x}$

divide equation by e^{3x}

$5C = 10$

$C = \frac{10}{5} = 2$

$C = 2e^{3x}$ P.I

G.S = $(C\cosh 2x + D\sinh 2x + 2e^{3x})$

(8) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1)\ 1(m+1) = 0$

$(m+1)$ twice

$y = e^{-x}(A+Bx)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$4Ce^{-2x} - 4Ce^{-2x} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

divide equation by e^{-2x}

$C = 1$

$C = e^{-2x}$

$g.s = e^{-x}(A+Bx) + e^{-2x}$

$$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$$

$$\frac{d^2y}{dx^2} + 25y = 0$$

$m^2 + 25 = 0$

$m^2 = -25$

$m = \pm j5$

$y = C\cos 5x + D\sin 5x$ C.F

$y = Cx^2 + Dx + E$

$$\frac{dy}{dx} = 2Cx + D$$

$$\frac{d^2y}{dx^2} = 2C$$

$2C + 25\left[x^2 + Dx + E\right] = 5x^2 + x$

$2C + 25Cx^2 + 25Dx + 25E = 5x^2 + x$

$25C = 5$

$$C = \frac{5}{25} = \frac{1}{5}$$

$25D = 1$

$$D = \frac{1}{25}$$

$2C + 25E = 0$

$2\left(\frac{1}{5}\right) + 25E = 0$

$2/5 + 25E = 0$

$25E = -2/5$

$E = -2/5 \times \frac{1}{25} = -2/125$

$y = 1/5\,x^2 + 1/25\,x - 2/125$

$R.S = C\cos 5x + D\sin 5x + 1/5\,x^2 + 1/25\,x - 2/125$

(5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$m^2 - 2m + 1 = 0$

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)$ twice

$m = 1$

$y = e^x (A+Bx)$ P.I

$y = (\cos x + D\sin x)$

$\frac{dy}{dx} = -C\sin x + D\cos x$

$\frac{d^2y}{dx^2} = -(\cos x - D\sin x$

$-(\cos x - D\sin x - 2[-(\sin x + D\sin x] + [\cos x + D\sin x = 4\sin x$

$-[\cos x - D\sin x + 2C\sin x - 2D\sin x + [\cos x + D\sin x = 4\sin x$

$-[\cos x - 2D\cos x + (\cos x - D\sin x + 2C\sin x + D\sin x = 4\sin x$

$\cos x[-C - 2D + C) + \sin x[-D + 2C + D) = 4\sin x$

$-C - 2D + C = 0 \Rightarrow 2D = 0 \quad D = 0$

$-D + 2C + D = 4$

$2C = 4 \Rightarrow C = \frac{4}{2} = 2$

$y = 2\cos x + D\sin x = 2\cos x$ P.I

G.S $= e^x (A+Bx) + 2\cos x$

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(6) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{5x}$

Given that $x=2$, $y=1$ and $\frac{dy}{dx} = -2$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

$m^2 + 4m + 5 = 0$

$a=1$, $b=4$ and $c=5$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{4^2 - 4\times1\times5}}{2\times1}$

$m = \frac{-4 \pm \sqrt{16-20}}{2}$

$m = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm \cancel{2}j}{\cancel{2}} = -2 \pm j$

$m_1 = -2+j \quad m_2 = -2-j$

$y = e^{-2x}(C\cos x + D\sin x)$

$y = Ce^{-2x}$

$\frac{dy}{dx} = -2Ce^{-2x}$

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$

$4Ce^{-2x} + 4[-2Ce^{-2x}] + 5[Ce^{-2x}] = 2e^{-3x}$

$4Ce^{-2x} - 3Ce^{-2x} + 5Ce^{-2x} = 2e^{-3x}$

$4C - 3C + 5x = 2$

$C = 2$

$y = 2e^{-3x}$

$y = e^{-2x}(C\cos x + D\sin\theta)$

at $x = 0$ and $y = 1$

$$1 = e^{-2(0)}\left[C\cos(0) + D\sin(0)\right] + 2e^{-2(0)}$$

$$1 = 1\left[C + 0\right] + 2$$

$$1 = C + 2$$

$$C = -2 + 1$$

$$C = -1$$

$$\frac{dy}{dx} = \left[e^{-2x}\left(-C\sin x + D\cos x\right)\right] + \left[2e^{-2x}\left(\cos x + D\sin x\right)\right] - 4e^{-2x}$$

When $\frac{dy}{dx} = -2$ $x = 0$

$$-2 = \left(D\right] + \left[-2C\right] - 4$$

$$-2 = D - 2C - 4$$

$$D - 2C = -2 + 4$$

$$D = -2 + 4 + 2(-1)$$

$$D = -2 + 4 - 2$$

$$D = 0$$

$$y = e^{-2x}\left(C\cos x + D\sin x\right) + 2e^{-2x}$$

$$y = e^{-2x}\left(-\cos x + \cancel{D\sin x}\right) + 2e^{-2x}$$

$$y = e^{-2x}\left(-\cos x + 2\right)$$

$$P.S = y = e^{-2x}\left(2 - \cos x\right)$$

(i) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$a = 3,\ b = -2$ and $c = -1$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 3 \times -1}}{2 \times 3}$

$m = \frac{2 \pm \sqrt{4 + 12}}{6}$

$m = \frac{2 \pm \sqrt{16}}{6}$

$m = \frac{\cancel{2} \pm 4}{\cancel{6}_3} \Rightarrow m = \frac{1 + 2}{3}$

$m = \frac{1+2}{3} = \frac{3}{3} = \frac{1}{1}$ or $m = \frac{1-2}{3} = \frac{-1}{3}$

$M_1 = \frac{1}{1}$ or $M_2 = \frac{-1}{3}$

$y = Ae^{1/1} + Be^{-1/3}$

$y = Cx + D$

$\frac{dy}{dx} = C$

$\frac{d^2y}{dx^2} = 0$

$3(0) - 2C - (Cx + D) = 2x - 3$

$-2C - Cx - D = 2x - 3$

$-C = 2$

$C = -2$

$-2C - D = -3$

$4 - D$

$D = 4 + 3 = 7$

G.S $= Ae^{1/1} + Be^{-1/3} - 2x + 7$

$y = 2x + \cancel{3}7$

(8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$

$m^2 - 6m + 8 = 0$

$m^2 - 2m - 4m + 8 = 0$

$m(m-2) - 4(m-2) = 0$

$(m-2)(m-4) = 0$

$m_1 = 2 \quad m_2 = 4$

$y = Ae^{2x} + Be^{4x}$

$y = Cxe^{4x}$

$\frac{dy}{dx} = 4Cxe^{4x} + Ce^{4x}$

$\frac{d^2y}{dx^2} = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x} - 24Cxe^{4x} - 6Ce^{4x} + 8Cxe^{4x} = 8e^{4x}$

$16Cxe^{4x} - 24Cxe^{4x} + 8Cxe^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$2Ce^{4x} = 8e^{4x}$

$2C = 8$

$C = \frac{8}{2} = 4$

$y = 4xe^{4x}$ P.I

G.S $= Ae^{2x} + Be^{4x} + 4xe^{4x}$