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① $\frac{dy}{dt} + 3y = e^{-2t}$ at $t = 0, y = 2$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s) = \frac{1}{(s+2)(s+3)} + \frac{2}{s+3}$

$\frac{1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{s+3}$

$1 = A(s+3) + B(s+2)$

when $s = -3$

$1 = -B$

$B = -1$

when $s = -2$

$1 = A$

$A = 1$

$\frac{1}{(s+2)(s+3)} = \frac{1}{s+2} - \frac{1}{s+3}$

$= L^{-1}\left[\frac{1}{s+2}\right] - L^{-1}\left[\frac{1}{s+3}\right] + L^{-1}\left[\frac{2}{s+3}\right]$

$= e^{-2t} - e^{-3t} + 2e^{-3t}$

② $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t = 0, y = 1$

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 6y(s) = \frac{2}{s^2+4} + 3$

$3y(s)[3s - 6] = \frac{2}{s^2+4} + 3$

$$y(s)(3s-6) = \frac{3s^2+14}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2+14 = A[s^2+4] + B[3s-6]$$

$$A = 3$$

$$4A - 6B = 14$$

$$-6B = 14-12$$

$$B = -\frac{1}{2}$$

$$y(s) = \frac{3}{3(s-2)} - \frac{1}{3}(s^2+4)$$

$$= e^{2t} - \frac{1}{6}\sin 2t$$

(3) $\frac{dy}{dt} - 4y = 8$, given that at $t=0$, $y=2$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$sy(s) - 2 - 4y(s) = \frac{8}{s}$$

$$sy(s) - 4y(s) = \frac{8}{s} + 2$$

$$y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + Bs$$

$$2(0)+8 = A(0-4)$$

$$A = -2$$

$$2(4)+8 = A(4-4) + B4$$

$$B = 4$$

$$\therefore L^{-1}\left[-\frac{2}{s} + \frac{4}{s-4}\right]$$

$$= -2 + 4e^{4t}$$

(4) $s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) = 2s - 3 + \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s(s-2) - 3(s-2) + 1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 4s - 3s + 6 + 1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s^2 - 2s + 5)(s-2)} = \frac{A}{(s-2)} + \frac{B}{(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$

$A = 2$

$-2A + B = -7$

$B = -7 + 4 = -3$

$= \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$

$= 2e^{2t} - \frac{1}{3} t \sin 2t$

(5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$s^2y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$

$(s^2 - 6s + 8)y(s) + [6 - s]y(0) - y'(0) = \frac{1}{s-3}$

$[s^2 - 6s + 8]y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3} = \frac{2s-5}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s - 5 = A(s-2)(s-4) + B(s-3)(s-4) + (s-3)(s-2)$

$2s - 5 = A(s^2 - 6s + 8) + B(s^2 - 7s + 12) + (s^2 - 5s + 6)$

$2(3) - 5 = 1 = A(3-2)(3-4) \quad -A = -1$

$2(4) - 5 = 3 = C(4-3)(4-2) \quad C = \frac{3}{2}$

$$-6A - 7B - 5C = 2$$

$$-6(-1) - 7(B) - 5\left(\frac{3}{2}\right) = 2$$

$$-7B = 2 + \frac{15}{2} - 6 = \frac{4+15-12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$L^{-1}\left[\frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{s(s-4)}\right] = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$