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DEPT: ELECT/ELECT ENGR.

ENG 881 Assign 4

$$1) (1-x^2) \frac{dy}{dx} - 2x \frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y' - 2xy' + 2y = 0$$

$$y^n = \frac{4n^2}{2!} + \frac{n(n-1)}{2!} x^2 + \dots$$

$$[y^{(2n)} - (1-x^2)y^{(2n)} + n y^{(2n)} + (-2x) \frac{n(n-1)}{2!} y^n + (-2)] + [y^{(2n)} \cdot (-2x)]$$

$$+ n y^{n-2} + [2y^n] = 0$$

$$(1-x^2)y^{(2n)} - 2xny^{(2n)} - n(n-1)y^{(2n)} - 2xy^{(2n)} - 2ny^{(2n)} + 2y^n = 0$$

let $x=0$

$$y^{(2n)} - n(n-1)y^{(2n)} - 2ny^{(2n)} + 2y^n = 0$$

$$y^{n+2} + y^n [-n(n-1) - 2n + 2] = 0$$

$$y^{n+2} + y^n [-n^2 + n - 2n + 2] = 0$$

$$y^{n+2} + y^n [-n^2 - n + 2] = 0$$

$$y^{n+2} = -y^n [-n^2 - n + 2]$$

$$n=0: y^2 = -y^0 \cdot 2 = -2y^0$$

$$n=1: y^3 = -y^1 [0] = 0$$

$$n=2: y^4 = -y^2 [-4] = 4y^2 = 4(-2y^0) = -8y^0$$

$$n=3: y^5 = -y^3 [-10] = 10y^3 = 10 \cdot 0 = 0$$

$$n=4: y^6 = -y^4 [-8] = 8y^4 = 8(-8y^0) = -64y^0$$

$$n=5: y^7 = -y^5 [-28] = 28[y^5]_0 = 28 \cdot 0 = 0$$

$$y = y^0 + \frac{x}{1!} y^1 + \frac{x^2}{2!} y^2 + \frac{x^3}{3!} y^3 + \dots$$

$$y = y^0 + \frac{x}{1!} y^1 + \frac{x^2}{2!} (-2)y^2 + \frac{x^3}{3!} (0) + \frac{x^4}{4!} (4)(-2)y^0 + \dots$$

$$+ \frac{x^5}{5!} (0) + \frac{x^6}{6!} (18) 4(-2)y^0 + \frac{x^7}{7!} (0)$$

$$y = y_0 + x y_1 - x^2 y_0 - \frac{x^4}{3x^1} y_0 - \frac{x^6}{5} y_0$$

$$y = y_0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} \right] + y_0 [x]$$

2. Transform each of the following function into Laplace (s) domain

i) $3e^{-4t} - 5e^{4t}$

Recall $L\{F(t) \pm G(t)\} = L\{F(t)\} \pm L\{G(t)\}$

$$= L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\} \Rightarrow 3L\{e^{-4t}\} - 5L\{e^{4t}\}$$

$$= 3 \cdot \left\{ \frac{1}{s-(-4)} \right\} - 5 \left\{ \frac{1}{s-4} \right\}$$

$$= 3 \cdot \left\{ \frac{1}{s - (-4)} \right\} - 5 \left\{ \frac{1}{s-4} \right\}$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$$\Rightarrow L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$$

$$= \frac{4}{s^2 + 4^2} + \frac{s}{s^2 + 4^2}$$

$$= \frac{4}{s^2 + 4^2} + \frac{s}{s^2 + 4^2}$$

$$= \frac{4}{s^2 + 16} + \frac{s}{s^2 + 16} = \frac{4 + s}{(s^2 + 16)}$$

iii) $t^3 + 2t^2 - t + 4$

$$\Rightarrow L\{t^3 + 2t^2 - t + 4\} = L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$$

$$t^n = \frac{n}{s^{n+1}}$$

$$= \frac{3!}{s^{3+1}} + 2 \left\{ \frac{2!}{s^{2+1}} \right\} - \left\{ \frac{1!}{s^{1+1}} \right\} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

IV. $e^{-5t} \cos 5t$

Recall the first shift theorem

$$L\{\cos 5t\} = \frac{s}{s^2 + 9}$$

$$= \frac{s}{s^2 + 5^2} = \frac{s}{s^2 + 25}$$

replacing s by a shift of $e^{-2t} : -s + 2$

$$\therefore L\{e^{-2t} \cos 5t\} = \frac{st^2}{[s+2]^2 + 25}$$

V. $t \sin 3t$

$$L\{\sin 3t\} = \frac{3}{s^2 + 9}$$

$$= \frac{3}{s^2 + 3^2}$$

$$f(s) = \frac{3}{s^2 + 9}$$

$$L\{t \sin 3t\} = -f'(s)$$

$f'(s)$ = using quotient rule

$$u = 3 \quad \frac{du}{ds} = 0$$

$$v = s^2 + 9 \quad \frac{dv}{ds} = 2s$$

$$= \frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2}$$

$$= \frac{[s^2 + 9] \cdot 0 - 3[2s]}{[s^2 + 9]^2}$$

$$= \frac{-6s}{[s^2 + 9]^2}$$

$$-F'(s) = -1 \cdot \left\{ \frac{-6s}{[s^2 + 9]^2} \right\}$$

$$= \frac{6s}{[s^2 + 9]^2}$$

$$vi) \frac{e^{-t} - e^{-2t}}{t}$$

$$L \left\{ \frac{e^{-t} - e^{-2t}}{t} \right\}$$

$$\lim_{t \rightarrow 0} \left\{ \frac{e^{-t} - e^{-2t}}{t} \right\} = \frac{1-1}{0} = \frac{0}{0} = \text{indeterminate}$$

Applying L'Hospital's rule

$$\lim_{t \rightarrow 0} \left\{ \frac{-e^{-t} - (-2)e^{-2t}}{1} \right\} = \left\{ \frac{-1+2}{1} \right\} = \frac{1}{1} = 1 \text{ (determinate)}$$

$$L \left\{ \frac{f(t)}{t} \right\} = \int_{0^+}^{\infty} f(s) ds$$

$$FG = L \{f(t)\}$$

$$L \{f(t)\} = e^{-t} - e^{-2t} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$FG = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\int FG = L \left\{ \frac{F(s)}{f} \right\} = \int_{s=5}^{\infty} \frac{1}{s+1} - \frac{1}{s+2} ds$$

$$= \int_{s=5}^{\infty} \frac{1}{s+1} ds - \int_{s=5}^{\infty} \frac{1}{s+2} ds$$

$$= [\ln(s+1) - \ln(s+2)]_s^{\infty}$$

$$= [\ln(s+1) - \ln(s+2)]_s^{\infty}$$

$$= \left[\frac{\ln(s+1)}{(s+2)} \right]_s^{\infty} = \ln \left[\frac{\infty+1}{(\infty+2)} - \frac{3+1}{(s+2)} \right]$$

$$= -\ln \left[\frac{s+1}{s+2} \right] = \ln \left[\frac{(s+2)}{s+1} \right]$$

vii) $e^{4t} \cos 2t$

$$L \{ e^{4t} \cos 2t \} = e^{4s} L \{ \cos 2t \}$$

$$L \{ \cos 2t \} = \frac{s}{s^2+2^2}$$

$$= \frac{s}{s^2+4}$$

replacing s by a shift of e^{4t} : $-s-4$

$$L \{ e^{4t} \cos 2t \} = \frac{s-4}{(s-4)^2+4}$$

viii) $t \sin 2t$

$$= L \{ t \sin 2t \} = -\frac{d}{ds} [F(s)]$$

$$F(s) = L \{ \sin 2t \} = \frac{2}{s^2+2^2}$$

$$F(s) = \frac{2}{s^2+4}$$

$F'(s)$ using quotient rule

$$u = 2$$

$$du/ds = 0$$

$$v = s^2+4$$

$$dv/ds = 2s$$

$$\frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2}$$

$$= \frac{(s^2+4) \cdot 0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$\therefore L \{ t \sin 2t \} = -F'(s)$$

$$= -L \left\{ \frac{-4s}{(s^2+4)^2} \right\}$$

$$= \frac{4s}{(s^2+4)^2}$$

$$ix) t^2 + 4t^2 + 5$$

$$L \{ t^2 + 4t^2 + 5 \} = L \{ t^2 \} + 4L \{ t^2 \} + L \{ 5 \}$$

$$= \frac{3!}{s^{3+1}} + 4 \left\{ \frac{2!}{s^{2+1}} \right\} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$x) e^{3t}(t^2+4)$$

$$\Rightarrow L \{ e^{3t} x \}$$

$$L \{ e^{3t} x \}$$

$$\begin{aligned} L\{x\} &= L\{t^2 + 4\} \\ &= L\{t^2\} + L\{4\} \\ &= \frac{2!}{s^2+1} + \frac{4}{s} \end{aligned}$$

$$= \frac{2}{s} + \frac{4}{s}$$

replacing s by a shift of $s-3$

$$L\{e^{3t} x\} = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

2) $L\{t^2 \cos t\}$

$$L\{t^2 \cos 3t\} = t^3 L\{\cos t\}$$

$$F(s) = L\{\cos t\} = \frac{s}{s^2+1^2}$$

$$F(s) = \frac{s}{s^2+1^2}$$

$F'(s)$ using quotient rule

$$u = s \quad \frac{du}{ds} = 1$$

$$v = s^2+1 \quad \frac{dv}{ds} = 2s$$

$$\frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2}$$

$$= \frac{[s^2+1] - 1 - 2s[s]}{[s^2+1^2]^2}$$

$$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2}$$

$$= \frac{-s^2+1}{[s^2+1^2]^2}$$

Recall

$$-f''(s) = -\frac{d}{ds} \left\{ \frac{s^2 - 1}{[s^2 + 1]^2} \right\}$$

using quotient rule

$$\frac{v \frac{du}{ds} - u \frac{dv}{ds}}{v^2}$$

$$u = s^2 - 1$$

$$\frac{du}{ds} = 2s$$

$$v = [s^2 + 1]^2$$

$$\frac{dv}{ds} = 4s[s^2 + 1]$$

$$\frac{[s^2 + 1]^2 \cdot 2s - [s^2 - 1][4s^2 + 4s]}{[s^2 + 1]^2}$$

$$= \frac{[2s^2 - 4s^2 + 2s] - [4s^2 - 4s]}{[s^2 + 1]^2}$$

$$= \frac{2s^2 - 4s^2 + 2s - 4s^2 + 4s}{[s^2 + 1]^2}$$

$$= \frac{-2s^2 - 4s^2 + 6s}{[s^2 + 1]^2}$$

$$= \frac{-2s^2 - 4s^2 + 6s}{s^4 + 2s^2 + 1}$$

$$f''(s) = -\frac{d}{ds} \left\{ \frac{-2s^2 - 4s^2 + 6s}{s^4 + 2s^2 + 1} \right\}$$

$$f''(s) = \frac{2s^2 + 4s^2 - 6s}{s^4 + 2s^2 + 1}$$

$$11) \sinh 2t$$

$$= L^{-1} \left\{ \frac{\sinh 2t}{t} \right\}$$

3) Convert the following function to time domains

$$1) \frac{s-5}{(s-3)(s-4)}$$

$$L^{-1} \left\{ \frac{s-5}{(s-3)(s-4)} \right\} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

Assuming $s=4$

$$4-5 = A(4-4) + B(4-3)$$

$$-1 = B(1)$$

$$B = -1$$

Assuming $s=3$

$$3-5 = A(3-4) + B(3-3)$$

$$-2 = A(-1)$$

$$A = 2$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2 \left\{ \frac{1}{s-3} \right\} - \left\{ \frac{1}{s-4} \right\}$$

$$= 2e^{3t} - e^{4t}$$

$$11) \frac{2s-6}{(s-2)(s-4)}$$

$$L^{-1} \left\{ \frac{2s-6}{(s-2)(s-4)} \right\}$$

$$2s-6 = A(s-4) + B(s-2)$$

Assuming $s=4$

$$(2(4)-6) = A(4-4) + B(4-2)$$

$$8-6 = A(0) + B(2)$$

$$2 = 2B$$

$$B = 1$$

Assuming $s=2$

$$2(2)-6 = A(2-4) + B(2-2)$$

$$4-6 = -2A+0$$

$$-2 = -2A$$

$$A = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1} \left\{ \frac{2s-6}{(s-2)(s-4)} \right\} = e^{2t} + e^{4t}$$

$$111) \frac{5s-8}{s(s-4)}$$

$$L^{-1} \left\{ \frac{5s-8}{s(s-4)} \right\} = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + B(s)$$

Assuming $s=4$

$$5(4)-8 = A(4-4) + B(4)$$

$$20-8 = 4B$$

$$12 = 4B$$

$$B = 3$$

Assuming $s \neq 0$

$$s(0) - 8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = 2$$

$$L^{-1} \left\{ \frac{5s - 8}{s(s-4)} \right\} = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3 \left\{ \frac{1}{s-4} \right\}$$

$$= 2 + 3e^{4t}$$

✓ $\frac{s-5}{s^2+4s+20}$

$$L \left\{ \frac{s-5}{s^2+4s+20} \right\} =$$

$$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$F(t) = e^{-2t} \cos 4t - \frac{7}{4} e^{-2t} \sin 4t$$

$$= e^{-2t} \left[\cos 4t - \frac{7}{4} \sin 4t \right]$$

$$\sqrt{\frac{s-5}{s^2+4s+20}}$$

$$\mathcal{L}^{-1} \left\{ \frac{s-5}{s^2+4s+20} \right\} =$$

$$f(s) = \frac{s-5}{s^2+4s+20}$$

$$V. \frac{s^2-3s-4}{(s-3)(s-1)^2}$$

$$= f(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A: \frac{s^2-3s-4}{(s-1)^2} \Big|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B: \frac{s^2-3s-4}{s-3} \Big|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$$C: \frac{d}{ds} \left[\frac{s^2-3s-4}{s-3} \right] \Big|_{s=1} = \frac{(s-3)(2s-3) - [s^2-3s-4]}{(s-3)^2}$$

$$\frac{(1-3)(2(1)-3) - [1^2-3(1)-4]}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3te^t + 2e^t$$

$$= e^t [3t+2] - e^{-3t}$$