

$$\textcircled{1} \quad \frac{dx}{dt} + 3y = e^{-2t} \quad \text{at } t=0, y=2$$

$$\frac{dx}{dt} = s(x) - y \cos$$

$$3y = 3(x)$$

$$e^{-it} = \frac{1}{s+2}$$

$$s(x) - y \cos + 3(x) = \frac{1}{s+2}$$

$$s(x) - 2 + 3(x) = \frac{1}{s+2}$$

$$x(s+3) = 2 + \frac{1}{s+2} = \frac{2(s+2)+1}{s+2}$$

$$x(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A(s+3) + B(s+2) = 2s+5$$

$$\text{at } s = -2$$

$$A(1) + 0 = -4 + 5 = 1$$

$$A = 1$$

$$\text{at } s = -3$$

$$0 + B(-1) = -6 + 5 = -1$$

$$B = 1$$

$$x(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y(t) = \mathcal{L}^{-1}(x(s)) = e^{-2t} + e^{-3t}$$

$$\textcircled{2} \quad 3 \frac{dy}{dt} - 6y = \sin 2t \quad \text{at } t=0, y=1$$

$$3 \frac{dy}{dt} = 3[s(x) - y \cos]$$

$$-6y = -6(x)$$

$$\sin 2t = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$3[s(x) - y \cos] - 6(x) = \frac{2}{s^2+4}$$

$$3s(x) - 3y \cos - 6(x) = \frac{2}{s^2+4}$$

$$3s(x) - 3(1) - 6(x) = \frac{2}{s^2+4}$$

$$3s(x) - 3 - 6(x) = \frac{2}{s^2+4}$$

$$x(s) [3s-6] = \frac{2}{s^2+4} + 3$$

$$x(s) [3s-6] = \frac{2+3(s^2+4)}{s^2+4} = \frac{3s^2+14}{s^2+4}$$

$$x(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$(s^2+4)(3s-6)$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A}{s^2+4} + \frac{B}{3s-6}$$

$$A(s^2+4) + B(3s-6) = 3s^2+14$$

$$3As^2 + 4A + 3Bs - 6B = 3s^2 + 14$$

$$3A + C = 3$$

$$3B - 6A = 0$$

$$4C - 6B = 14$$

$$C = \frac{3}{3A} \quad 3 - 3A$$

$$4(3-3A) - 6B = 14$$

$$12 - 12A - 6B = 14$$

$$12A + 6B = -2$$

$$6A + 3B = -1$$

$$6A = -1 - 3B$$

$$3B - (-1 - 3B) = 0$$

$$3B + 1 + 3B = 0$$

$$6B = -1$$

$$B = -\frac{1}{6}$$

$$6A = -1 - 3(-\frac{1}{6}) = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$A = -\frac{1}{2} \div 6 = -\frac{1}{12}$$

$$3A + C = 3$$

$$C = 3 - 3(-\frac{1}{12}) = 3 + \frac{1}{4} = \frac{13}{4}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{-\frac{1}{12}s}{s^2+4} - \frac{1}{6} + \frac{13}{4(3s-6)}$$

$$= \frac{-\frac{1}{12}s}{s^2+4} - \frac{1}{6} + \frac{13}{12(s-2)}$$

$$= \frac{-\frac{1}{12}s}{s^2+4} - \frac{1}{6} + \frac{13}{12(s-2)}$$

$$y(t) = \frac{1}{12} (\cos t - \frac{1}{2} \sin t) + \frac{13}{12} e^{2t}$$

$$y(t) = \frac{1}{12} (\cos t + \sin t - 13 e^{2t})$$

$$\frac{dy}{dt} - 4y = 8$$

$$\text{at } t=0, y = -2$$

$$\frac{dy}{dt} = sY(s) - y(0)$$

$$-4y = 8 \Rightarrow 4Y(s)$$

$$8 = \frac{8}{s^2}$$

$$sY(s) - y(0) - 4Y(s) = \frac{8}{s^2}$$

$$sY(s) - 2 - 4Y(s) = \frac{8}{s^2}$$

$$Y(s) [s-4] = \frac{8}{s^2} + 2 = \frac{8+2s^2}{s^2}$$

$$Y(s) = \frac{2s^2+8}{s^2(s-4)} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s^2}$$

$$A(s)(s-4) + B(s-4) + C(s^2) = 2s^2+8$$

$$\text{at } s=0$$

$$0A - 4B + 0 = 8$$

$$B = -2$$

$$\text{at } s=4$$

$$0 + 0 + 16C = 40$$

$$C = \frac{5}{4}$$

$$As^2 + Bs^2 = 2s^2$$

$$A+B = 2$$

$$A-2 = 2$$

$$A = 4$$

$$Y(s) = \frac{4}{s} - \frac{2}{s-4} + \frac{5}{s^2}$$

$$\textcircled{A} \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t} \quad \text{at } t=0, y=2, y'=1$$

$$\frac{d^2y}{dt^2} = s^2 Y(s) - sY(s) - y'$$

$$-2\frac{dy}{dt} = -2[sY(s) - y(0)]$$

$$5y = 5Y(s)$$

$$e^{2t} = \frac{1}{s-2}$$

$$s^2 Y(s) - sY(s) - y' - 2[sY(s) - y(0)] + 5Y(s) = \frac{1}{s-2}$$

$$= \frac{1}{s-2}$$

$$s^2 Y(s) - 2s - 1 - 2sY(s) + 4 + 5Y(s) = \frac{1}{s-2}$$

$$Y(s) [s^2 - 2s + 5] = \frac{1}{s-2} + 2s - 3$$

$$Y(s) [s^2 - 2s + 1 + 4] = \frac{1+2s(s-2)-3(s-2)}{s-2}$$

$$Y(s) = \frac{1+2s^2-4s-3s+6}{(s-2)4} = \frac{s^2-2s+5}{(s-2)4}$$

$$Y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$$

$$(s-2)(s^2-2s+5)$$

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$$d^2y/dt^2 - 6dy/dt + 8y = e^{3t} \quad \text{at } t=0, y=0, y'=2$$

$$d^2y/dt^2 = s^2 Y(s) - sY(s) - y'$$

$$-6dy/dt = -6[sY(s) - y(0)]$$

$$8y = 8Y(s)$$

$$e^{3t} = 1/s-3$$

$$s^2 Y(s) - sY(s) - y' - 6sY(s) + 6y(0) + 8Y(s) = 1/s-3$$

$$\text{at } t=0, y=0, y'=2$$

$$s^2 Y(s) - 2 - 6sY(s) + 8Y(s) = 1/s-3$$

$$Y(s) [s^2 - 6s + 8] = 1/s-3 + 2 = \frac{1+2(s-3)}{s-3} = \frac{2s-5}{s-3}$$

$$Y(s) (s-4)(s-2) = \frac{2s-5}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4) = 2s-5$$

$$\text{at } s=2$$

$$C(-1)(-2) = 4-5$$

$$2C = -1$$

$$C = -1/2$$

$$\text{at } s=3$$

$$A(-1)(1) = 2(3)-5 = 1$$

$$A = -1$$

$$\text{at } s=4$$

$$B(1)(2) = 2(4)-5 = 8-5 = 3$$

$$B = I$$

$$Y(s) = \frac{-1}{s-3} + \frac{1}{s-4} - \frac{1}{2} \cdot \frac{1}{s-2}$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = -e^{3t} + e^{4t} - \frac{1}{2}e^{2t}$$