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Mechatronics
15/ENG05/015

1. $\frac{d^2y}{dt^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

$m^2 + 4m + 5 = 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2j}{2} = -2 \pm j$

C.F $y = e^{-2\theta}(C\cos\theta + D\sin\theta)$

P.I $y = C\cos kx + D\sin kx$

$y = C\cos\theta + D\sin\theta \Rightarrow \frac{dy}{d\theta} = -C\sin\theta + D\cos\theta$

$(-C\cos\theta - D\sin\theta) + 4(-C\sin\theta + D\cos\theta) + 5(C\cos\theta + D\sin\theta) = 6\sin\theta$

$-C\cos\theta - D\sin\theta - 4C\sin\theta + 4D\cos\theta + 5C\cos\theta + 5D\sin\theta = 6\sin\theta$

$(-C + 4D + 5C)\cos\theta + (-D - 4C + 5D)\sin\theta = 6\sin\theta$

$4C + 4D = 0,\ 4C = -4D,\ C = -D \Rightarrow 4D - 4C = 6$

$4D - 4(-D) = 6 \Rightarrow 4D + 4D = 6 \Rightarrow 8D = 6 \quad \therefore D = 3/4$

$4C + 4D = 0 \Rightarrow 4C + 4(3/4) = 0 \Rightarrow 4C + 3 = 0$

$4C = -3 \quad \therefore C = -3/4$

P.I $= y = -3/4\cos\theta + 3/4\sin\theta$

General Solution

$y = e^{-2\theta}(C\cos\theta + D\sin\theta) - 3/4\cos\theta + 3/4\sin\theta$

ii) at $\theta = \infty$ and $dx/dt = 0$ or $dy/d\theta = 0$

$\frac{dy}{d\theta} = (e^{-2\theta})(-C\sin\theta + D\cos\theta) + (C\cos\theta + D\sin\theta)(-2e^{-2\theta}) + 3/4\sin\theta + 3/4\cos\theta$

$\frac{dy}{d\theta} = (e^{-2\theta})(D\cos\theta - C\sin\theta) - 2e^{-2\theta}(C\cos\theta + D\sin\theta) + 3/4\sin\theta + 3/4\cos\theta$

at $\theta = \infty$

$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$-3/4\sin\theta = 3/4\cos\theta \Rightarrow -\sin\theta = \cos\theta$

$\frac{-\sin\theta}{\cos\theta} = 1 \Rightarrow -\tan\theta = 1 \Rightarrow \tan\theta = -1$

$\theta = \tan^{-1} -1 \quad \therefore \theta = -45^\circ$

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(2) $EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$

$EIm^2 = 0 \Rightarrow m^2 = 0 \Rightarrow m = \pm 0$

$y = e^{0x}(A + Bx) \Rightarrow y = A + Bx$

P.I $y = Fx^2 + Gx^3 + Hx^4 \Rightarrow$

$\frac{dy}{dx} = 2Fx + 3Gx^2 + 4Hx^3 \Rightarrow \frac{d^2y}{dx} = 2F + 6Gx + 12Hx^2$

$EI(2F + 6Gx + 12Hx^2) = \frac{w}{2}(L-x)^2 \Rightarrow 2FEI + 6GEIx + 12HEIx^2 = \frac{w}{2}(L-x^2)$

$4FEI + 12GEIx + 24HEIx^2 = w(L^2 - 2Lx + x^2) \Rightarrow 4FEI + 12GEIx + 24HEIx^2 = wL^2 - 2wLx + wx^2$

$24HEI = w \Rightarrow H = \frac{w}{24EI} \Rightarrow 12GEI = -2wL$

$G = \frac{-2wL}{12EI} = \frac{-wL}{6EI} \Rightarrow 4FEI = wL^2 \Rightarrow F = \frac{wL^2}{4EI}$

$y = \left(\frac{wL^2}{4EI}\right)x^2 - \left(\frac{wL}{6EI}\right)x^3 + \left(\frac{w}{24EI}\right)x^4$

$y = \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$ P.I $y = \frac{w}{24EI}\left(6L^2w^2 - 4Lx^3 + x^4\right)$

General Solution

$y = A + Bx + \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$

at $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$

$0 = A$

$\frac{dy}{dx} = B + \frac{w}{24EI}\left(12L^2x - 12Lx^2 + 4x^3\right)$

$0 = B$

Particular solution

$y = \frac{w}{24EI}\left(6L^2x^2 - 4Lx^3 + x^4\right)$

$y = \frac{wx^2}{24EI}\left(6L^2 - 4Lx + x^2\right)$

when $x = L \Rightarrow y = \frac{wL^2}{24EI}\left(6L^2 - 4L^2 + L^2\right)$

$y = \frac{wL^4}{24EI}(3)$

$y = \frac{wL^4}{8EI}$