Name: Chinda Precious .K.
matric no: 15/ENG07/011
department: Petroleum

A ssignment.

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

$[y^{(2+n)} \cdot (1-x^2) + n y^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!} y^n (-2)] + [y^{(1+n)} \cdot 2x + n y^n \cdot 2 + [2y^n] = 0$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

Let $x = 0$

$y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n[-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n)_0[-n^2 - n + 2]$

$n = 0:\ y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1:\ y^3 = -y' \cdot [0] = 0$

$n = 2:\ y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3:\ y^5 = -y^3 \cdot [-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4:\ y^6 = -y^4[-18] = 10y^4 = 18 \cdot 4 \cdot -2 \cdot y^0$

$n = 5:\ y^7 = -y^5[-28] = 28(y^5)^0 = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2 y^3}{2!} + \frac{x^3}{3!}y^3 + \cdots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x}{4!}(4)(-2)y^0 + -$

$+ \frac{x^5}{5!}(0) + \frac{x^3}{6!}(16)^4(-2)y^0 + \frac{x^9}{7!}(0)$

$y = y' + xy' - x^2 y^0 - \frac{x^4}{3\times1} y^0 - \frac{x^6}{5} y^0$

$y = y^0 [1 - x^2 - \frac{x^4}{3} - \cancel{\frac{x^6}{5}} \frac{x^6}{5}] + y'_0 [x]$

2i) $3e^{-4t} - 5e^{4t}$

$= L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$

$= 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii] $\sin 4t + \cos 4t$

$L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$

$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$

iii] $t^2 + 2t^2 - t + 4$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t} \cos 5t$

$L[\cos 5t] = \frac{s}{s^2+a}$

$= \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$

$$L[e^{-2t}\cos 5t] = \frac{s+2}{[s+2]^{2}+25}$$

v) $t\sin 3t$

$$L[\sin 3t] = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2} = \frac{3}{s^2+a}$$

$$L[t\sin 3t] = -F'(s) \quad \frac{v\,du/ds - u\,dv/ds}{v^2}$$

$u=3 \qquad du/ds=0$

$v=s^2+a \qquad dv/ds=2s$

$$= \frac{[s^2+a]\,0-3[2s]}{[s^2+a]^2}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$$-F'(s) = -1\left[\frac{-6s}{[s^2+a]^2}\right]$$

$$= \frac{6s}{[s^2+a]^2}$$

vi) $\dfrac{e^{-t}-e^{-2t}}{t}$

$$L\left[\frac{e^{-t}-e^{-2t}}{t}\right]$$

$$L[f(t)] = e^{-t}-e^{-2t} - \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F\sigma = \frac{1}{\sigma+1}\ \frac{-1}{\sigma+2}$$

$$\int F\sigma\, L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$$

$$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1}\, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$$

$$\ln[\sigma+1] - \ln[\sigma+2]\Big]_s^{\infty}$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+1)\right]_s^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

vii) $e^{4t}\cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{[s-4]^2+4}$$

viii) $t\sin 2t$

$$L[t\sin 2t] = \frac{-d}{ds}[F(s)]$$

$$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$F(s) = \frac{2}{s^2+4}$$

$u = 2$ $\qquad du/ds = 0$

$v = s^2+4$ $\qquad dv/ds = 2s$

$$\frac{v\frac{du}{ds} - v\, du/ds}{v^2}$$

$$= \frac{(s^2+4)\,0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$L[t\sin 2t] = F'(s)$$

$$= -1.\left[\frac{4s}{(s^2+4)^2}\right]$$

$$= \frac{4s}{(s^2+4)^2}$$

ix) $t^3 + 4t^2 + 5$

$L[t^3 + 4t^2 + 5]$

$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $e^{3t}(t^2 + 4)$

$\text{let } x = t^2 + 4$

$L[e^{3t}x]$

$L[x] = L[t^2 + 4]$

$= L[t^2] + L[4]$

$= \frac{2!}{s^{2+1}} + \frac{4}{s}$

$= \frac{2}{s^3} + \frac{4}{s}$

$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$

xi) $t^2 \cos t$

$L[t^2 \cos t] = t^2 L[\cos t]$

$F(s) = L[\cos t] = \frac{s}{s^2 + 1^2}$

$F(s) = \frac{s}{s^2 + 1^2}$

$F'(s) =$ $\quad du/ds = 1$

$u = s$ $\quad dv/ds = 2s$

$v = s^2 + 1^2$

$= \frac{[s^2 + 1^2]1 - 2s[s]}{[s^2 + 1^2]^2}$

$= \frac{s^2 + 1^2 - 2s^2}{[s^2 + 1^2]^2} = \frac{-s^2 + 1}{[s^2 + 1^2]^2}$

$$-f''(s) = \frac{-d}{ds}\left[\frac{s^2-1}{[s^2+1^2]^2}\right]$$

$u = s^2-1 \qquad du/ds = 2s$

$v = [s^2+1]^2 \quad dv/ds = 4s[s^2+1]$

$$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^3+4s]}{[s^2+1]^2}$$

$$= \frac{[2s^5-4s^3+2s]-[4s^5-4s]}{[(s^2+1)^2]}$$

$$= \frac{2s^5-4s^3+2s-4s^5+4s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5-4s^3+6s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5-4s^3+6s}{s^4+2s^2+1}$$

$$f''(s) = \frac{-d}{ds}\left[\frac{-2s^5-4s^3+6s}{s^4+2s^2+1}\right]$$

$$f''(s) = \frac{2s^5+4s^3-6s}{s^4+2s^2+1}$$