

DATE: \_\_\_\_\_  
 16/09/2021  
 Page No. \_\_\_\_\_

$$3 \frac{dy}{dx} + y = e^{-2x}$$

$$y' + y = e^{-2x}$$

$$y(x) = \frac{1}{s+2}$$

$$y(x) [s+2] = \frac{1}{s+2}$$

$$[s+2] y(x) = \frac{1}{s+2}$$

$$y(x) [s+2] = \frac{1}{s+2}$$

$$y(x) = \frac{1 + 2s + 4}{(s+2)(s+2)}$$

$$y(x) = \frac{2s+5}{(s+2)(s+2)} = \frac{A}{s+2} + \frac{B}{s+2}$$

$$2s+5 = A(s+2) + B(s+2)$$

$$s = -2$$

$$-1 = -B$$

$$B = 1$$

$$s = -2$$

$$1 = A$$

$$= \frac{1}{s+2} + \frac{1}{s+2}$$

$$y(x) = e^{-2x} + e^{-2x}$$

$$ii) 8 \frac{dy}{dx} - 6y = \cos t + 100, y=1$$

$$8 y' - 6y = \cos t$$

$$8[sy(x) - y(x)] = 6 - y(x) = \frac{2}{s^2+4}$$

$$8s y(x) - 6y(x) = 6 - y(x) = \frac{2}{s^2+4}$$

$$y(x) [8s-6] = \frac{2}{s^2+4}$$

$$y(x) [8s-6] = \frac{2}{s^2+4} + 9$$

$$y(x) = \frac{2 + 9(s^2+4)}{(s^2+4)(8s-6)}$$

$$= \frac{2 + 36s^2 + 36}{(s^2+4)(8s-6)}$$

$$= \frac{36s^2 + 38}{(s^2+4)(8s-6)} = \frac{A+Bs + C}{s^2+4} + \frac{D}{8s-6}$$

$$36s^2 + 38 = A + B(8s-6) + C(s^2+4)$$

$$s = 0$$

$$26 = C[4+4]$$

$$C = \frac{26}{8} = \frac{13}{4}$$

Coefficient of  $s^2$

$$3 = 8B + C$$

$$\frac{3-C}{8} = B$$

$$B = \frac{3 - \frac{13}{4}}{8} = \frac{-1}{12}$$

Coefficient of  $s$

$$0 = 8A - 6B$$

$$A = \frac{6B}{8} = 2B = 2 \times \frac{-1}{12}$$

$$= \frac{-1}{6}$$

$$y(x) = \frac{-1/6 - 1/12 s}{s^2+4} + \frac{13}{4} \times \frac{1}{8s-6}$$

$$y(x) = \frac{-2-3}{s^2+4} + 1$$

$$y(x) = \frac{-1}{6} \times \frac{1}{s^2+4} + \frac{13}{4} \times \frac{1}{8s-6}$$

$$+ \frac{13}{4} \frac{1}{8(s-2)}$$

$$y(x) = \frac{-1 \times 2}{10 \times 5 \times 4} + \frac{-1 \times 3}{10 \times 5 \times 4} + \frac{19 \times 1}{10 \times 5 \times 2}$$

$$y(x) = -\frac{1}{10} \sin x - \frac{1}{10} \cos x + \frac{19}{10} e^{2x}$$

$$y(x) = \frac{1}{5} [\sin x + \cos x - 19e^{2x}]$$

$$ii) \frac{dy}{dx} - 4y = 8, \quad y=0, \quad y=3$$

$$y' - 4y = 8$$

$$y'(x) - y(x) - 4y(x) = 8/5$$

$$y'(x) (3-x) + 2 = 8/5$$

$$y'(x) = \frac{8}{5} + 2 = \frac{18}{5}$$

$$y(x) = \frac{8+25}{5} = \frac{33}{5}$$

$$= \frac{8+25}{5} = \frac{A}{5} + \frac{B}{3-4}$$

$$8+25 = A(3-4) + B(5)$$

$$4 = 4$$

$$8+25(4) = 4B$$

$$8+100 = 4B$$

$$4B = 108$$

$$B = 27$$

$$3 = 0$$

$$8 = -4A$$

$$A = -2$$

$$y(x) = \frac{4}{5} - \frac{2}{3}$$

$$y(x) = \frac{4e^{4x} - 2}{5}$$

$$ii) \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}, \quad y(0) = 1, \quad y(\pi) = 1$$

$$y'' - 2y' + 5y = e^{2x}$$

$$y''(x) - y'(x) - y'(x) - 2y(x) + 5y(x)$$

$$+ 4y(x) = 1/5 \cdot 5$$

$$y'(x) - 5y(x) - 1 - 2y(x) + 2(x) + 5y(x)$$

$$y(x) = 1/5 \cdot 2$$

$$y(x) = [5^2 - 2 \cdot 2 + 5] - 25 - 1 + 4 = 1/5$$

$$y(x) [5^2 - 2 \cdot 2 + 5] = \frac{1}{5} - 5 + 25$$

$$y(x) [5^2 - 2 \cdot 2 + 5] = \frac{1}{5} (5^2 - 2 \cdot 2 + 5)$$

$$y(x) [5^2 - 2 \cdot 2 + 5] = \frac{1 - 25 - 0 + 25^2}{5 \cdot 2}$$

$$y(x) [5^2 - 2 \cdot 2 + 5] = \frac{1 - 25 - 0 + 25^2}{5 \cdot 2}$$

$$y(x) = \frac{2 \cdot 5^2 + 5 + 7}{(5-2)(5^2 - 2 \cdot 2 + 5)}$$

$$y(x) = \frac{A}{5-2} + \frac{B \cdot C}{5^2 - 2 \cdot 2 + 5}$$

$$25^2 - 25 - 1 = A(5^2 - 2 \cdot 2 + 5) + [5 \cdot 7 + 2](5-2)$$

$$25^2 - 25 - 1 = A(25 - 4 + 5) + [35 + 2](3)$$

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$$25^2 - 25 - 1 = A(25 - 4 + 5) + [35 + 2](3)$$

$$25^2 - 25 - 1 = A(25 - 4 + 5) + [35 + 2](3)$$

$$y(s) = \frac{1}{s-3} + \frac{9s-9}{s^2-6s+9}$$

$$= \frac{1}{s-3} + \frac{9}{s^2-6s+9} - \frac{9}{s^2-6s+9}$$

$$= \frac{1}{s-3} + \frac{9}{5(s-3)^2} - \frac{3}{(s-3)^2+2^2}$$

$$= \frac{1}{s-3} + \frac{9}{5} \frac{1}{s-3} - \frac{1}{(s-3)^2+2^2} - \frac{3}{2}$$

$$\frac{2}{(s-3)^2+2^2}$$

$$j(t) = \frac{1}{5} e^{3t} + \frac{9}{5} e^{3t} \cos t + \frac{1}{2} e^{3t} \sin t$$

$$= \frac{2}{5} e^{3t} \cos t$$

$$j(t) = \frac{1}{5} e^{3t} + \frac{9}{5} e^{3t} \cos t - 2 e^{3t} \sin t$$

$$\left[ \frac{dy}{dt} - 3y = 1 \right] \Rightarrow y = e^{3t}, t=0, j(0), j'(0)$$

$$y'' - 6y' + 9y = e^{3t}$$

$$y''(0) - 6y'(0) + 9y(0) = 1$$

$$9y(0) - 0 - 6 = 6y'(0) + 0 + 9y(0) = 1$$

$$j(s) [s^2 - 6s + 9] = \frac{1}{s-3} - 12$$

$$j(s) = \frac{1 + 2(s-3)}{s-3} - \frac{12}{s^2-6s+9}$$

$$j(s) = \frac{1 + 2s - 6}{(s-3)(s^2-6s+9)}$$

$$j(s) = \frac{2s-5}{(s-3)(s^2-6s+9)}$$

$$j(s) [s^2 - 6s + 9] = \frac{1}{s-3} - 12$$

$$j(s) = \frac{1 + 2(s-3)}{s-3} - \frac{12}{s^2-6s+9}$$

$$j(s) = \frac{1 + 2s - 6}{(s-3)(s^2-6s+9)}$$

$$j(s) = \frac{2s-5}{(s-3)(s^2-6s+9)}$$

$$j(s) = \frac{2s-5}{(s-3)(s-3)(s-3)} = \frac{A}{s-3} + \frac{B}{s-3} + \frac{C}{s-3}$$

for A

$$\frac{2s-5}{(s-3)(s-3)(s-3)} = \frac{A}{s-3} + \frac{B}{s-3} + \frac{C}{s-3}$$

$$= -1$$

for B

$$\frac{2s-5}{(s-3)(s-3)(s-3)} = \frac{A}{s-3} + \frac{B}{s-3} + \frac{C}{s-3}$$

$$= \frac{3}{2}$$

for C

$$\frac{2s-5}{(s-3)(s-3)(s-3)} = \frac{A}{s-3} + \frac{B}{s-3} + \frac{C}{s-3}$$

$$= -\frac{1}{2}$$

$$j(s) = \frac{1}{s-3} + \frac{3}{2} \times \frac{1}{s-3} - \frac{1}{2} \times \frac{1}{s-3}$$

$$= e^{3t} + \frac{3}{2} e^{3t} - \frac{1}{2} e^{3t}$$