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15/ENG03/002
Civil Engineering

1) $\frac{dy}{dt} + 3y = e^{-2t}$

$t = 0 \quad y = 2$

$sy(s) + y(s) + 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) + 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$L^{-1}\left\{\frac{2s+5}{(s+2)(s+3)}\right\} = y(s)$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$2s + 5 = A(s+3) + B(s+2)$

when $s = -2$

$-4 + 5 = A(1) + 0$

$A = 1$

when $s = -3$

$-6 + 5 = A(0) + B(1)$

$-B = -1$

$B = 1$

$y(s) = L^{-1}\left[\frac{1}{s+2} - \frac{1}{s+3}\right]$

$y(s)\, e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$

at $t = 0 \quad y = 1$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$$

$$3y(s)(s-2) - 3 = \frac{2}{s^2+4}$$

$$3y(s)(s-2) = \frac{2}{s^2+4} + 3$$

$$\frac{2 + 3s^2 + 12}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{3(s-2)(s^2+4)} = \frac{3s^2+14}{(3s-6)(s^2+4)}$$

$$y(s) = L^{-1}\left(\frac{3s^2+14}{(3s-6)(s^2+4)}\right)$$

$$\frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}$$

$$3s^2+14 = A(s^2+4) + Bs + C(3s-6)$$

at $s = 2$

$$12+14 = A(8) + (Bs+C)(0)$$

$$26 = 8A$$

$$A = 13/4$$

Using the method of Coefficient

$$3 = A + B \quad --- \quad 1$$

$$3 = 13/4 + 3B$$

$$3B = 3 - 13/4$$

$$3B = -1/4 \qquad B = -1/12$$

$$14 = 4A - 6C$$

$$14 = 4 \times \frac{13}{4} - 6C$$

$$14 = 13 - 6C$$

$$1 = -6C$$

$$C = 1/6$$

$$y(s) = L^{-1}\frac{13}{14} \times \frac{1}{3s-6} + \frac{(-1/12 s - 1/6)}{s^2+4}$$

$$L^{-1}\left\{\frac{13\times 1}{12\times s-2} - \frac{1}{12}\times\frac{1}{s^2+4} - \frac{1}{6}\times\frac{1}{s^2+4}\right\}$$

$$= L^{-1}\left(\frac{13}{12}\times\frac{1}{s-2}\right) + L^{-1}\left(\frac{1}{12}\times\frac{s}{s^2+4}\right) +$$

$$L^{-1}\left(\frac{1}{6\times 2}\times\frac{2}{s^2+4}\right)$$

$$= \frac{13}{12}e^{2t} - \frac{1}{12}\cos 2t \cdot \frac{1}{12}\sin 2t$$

(3) $\frac{dy}{dt} - 4y = 8$

at $t=0$, $y=2$

$3y(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 2 - 4y(s) = 8/s$

$y(s)(s-4) = 8/s + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$y(s) = L^{-1}\frac{8+2s}{s(s-4)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A(s-4)+B(s)}{s(s-4)}$$

$8+2s = A(s-4) + B(s)$

when $s=4$

$8+8 = A(0) + 4B$

$B = 4$

when $s=0$

$8 = -4A + 0$

$-4A = 8$

$A = -2$

$y(s) = L^{-1}\left(\frac{-2}{s} + \frac{4}{s-4}\right)$

$y(s) = -2 + 4e^{4t}$

(N) $\dfrac{d^2y}{dt^2} - 2\dfrac{dy}{dt} + 5y = e^{2t}$

$s^2 y(s) - s y(0) - y'(0) - 2(s y(0) - y(0)) + 5y$
$= \dfrac{1}{s-2}$

at $t = 0$ $y = 2$ $y' = 1$

$s^2 y(s) - s(2) - 1 - 2(s y(s) - 2) + 5y(s)$

$y(s)(s^2 - 2s + 5) - 2s + 3 = 1/s - 2$

$y(s)(s^2 - 2s + 5) = \dfrac{1}{s-2} + \dfrac{2s}{1} + \dfrac{3}{1}$

$\dfrac{1 + 2s^2 - 4 + 3s - 6}{s-2}$

$y(s) s^2 - 2s + 5 = \dfrac{2s^2 + 3s - 7}{s-2}$

~~Us~~: $2s^2 + 3s - 9 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$a + s = 2$

$8 + 6 - 9 = A(4 - 8 + 5) + 0$

$A = 5$

Using the method of coefficient

$2 = A + B$ $2 = 5 + B$ $B = -3$ $C = -7$

$y(s) = \left(\dfrac{5}{s-2} + \dfrac{(-3s-7)}{s^2 - 2s + 5}\right)$

$L^{-1}\left(\dfrac{5}{s-2}\right) - L^{-1}\left[3 \times \dfrac{s}{(s-1)^2 + 4}\right] - L^{-1}\left(\dfrac{7}{2} \times \dfrac{2}{(s-1)^2 + 4}\right)$

$= 5e^{2t} - 3e^{t}\cos 2t - 7/2\, e^{t} \sin 2t.$

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