# Assignment 5

i) $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0$ $y=2$

$y' + 3y = e^{-2t}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] - 2 = \frac{1}{s+2}$

$(s+3)y(s) = \frac{1}{s+2} + 2$

$y(s)[s+3] = \frac{1+2(s+2)}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)} \equiv \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

$s = -3$

$-1 = -B$

$B = 1$

$s = -2$

$1 = A$

$= \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

ii) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$

$3y' - 6y = \sin 2t$

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3As - 6A + 3Bs^2 - 6Bs$

$$y(s)[3s-6] - 3 = \frac{2}{s^2+4}$$

$$y(s)[3s-6] = \frac{2}{s^2+4} + 3$$

$$y(s) = \frac{2+3(s^2+4)}{(s^3+4)(3s-6)}$$

$$= \frac{2+3s^2+12}{(s^2+4)(3s-6)}$$

$$= \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{s^2+4} + \frac{C}{3s-6}$$

$$3s^2+14 = A+Bs(3s-6) + C(s^2+4)$$

$$s = 2$$

$$26 = C(4+4)$$

$$C = \frac{26}{8} = \frac{13}{4}$$

Coefficient of $s^2$

$$3 = 3B + C$$

$$\frac{3-C}{3} = B$$

$$B = \frac{3 - \frac{13}{4}}{3} = \frac{-1}{12}$$

Coefficient of $s$

$$0 = 3A - 6B$$

$$A = \frac{6B}{3} = 2B = 2 \times \frac{-1}{12}$$

$$= \frac{-1}{6}$$

$$y(s) = \frac{\frac{-1}{6} - \frac{1}{12}s}{s^2+4} + \frac{13}{4} \cdot \frac{1}{3s-6}$$

$$y(s) = \frac{-2-s}{12} \times \frac{12}{s^2+4} + \frac{1}{4} \cdot \frac{1}{3(s-2)}$$

$$-\frac{1}{6} \cdot \frac{1}{s^2+4} - \frac{1}{12}s \cdot \frac{1}{s^2+4}$$

$$+ \frac{13}{4} \frac{1}{3(s-2)}$$

$$y(s) = \frac{-1}{12}\cdot\frac{2}{s^2+4} - \frac{1}{12}\cdot\frac{s}{s^2+4} + \frac{13}{12}\cdot\frac{1}{s-2}$$

$$y(t) = \frac{-1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

$$y(t) = \frac{-1}{12}\left[\sin 2t + \cos 2t - 13e^{2t}\right]$$

iii) $\frac{dy}{dt} - 4y = 8 \quad t=0 \quad y=2$

$y' - 4y = 8$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$y(s)[s-4] - 2 = \frac{8}{s}$

$y(s) = \frac{8}{s} + 2\cdot\frac{1}{s-4}$

$y(s) = \frac{8+2s}{s}\cdot\frac{1}{s-4}$

$= \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8+2s = A(s-4) + Bs$

$s = 4$

$8+2(4) = 4B$

$8+8 = 4B$

$4B = 16$

$B = 4$

$s = 0$

$8 = -4A$

$A = -2$

$y(s) = \frac{4}{s-4} - \frac{2}{s}$

$y(t) = 4e^{4t} - 2$

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ $\quad t = 0 \quad y = 2 \quad y' = 1$

$y'' - 2y' + 5y = e^{2t}$

$s^2Y(s) - sy(0) - y'(0) - 2sY(s) + 2y(0) + 5(Y(s)) = \frac{1}{s-2}$

$s^2Y(s) - 2s - 1 - 2sY(s) + 4 + 5Y(s) = \frac{1}{s-2}$

$Y(s)[s^2 - 2s + 5] - 2s - 1 + 4 = \frac{1}{s-2}$

$Y(s)[s^2 - 2s + 5] = \frac{1}{s-2} - 3 + 2s$

$Y(s)[s^2 - 2s + 5] = \frac{1 - 3(s-2) + 2s(s-2)}{s-2}$

$Y(s)[s^2 - 2s + 5] = \frac{1 - 3s + 6 + 2s^2 - 4s}{s-2}$

$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$

$Y(s) = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$s = 2$

$8 - 14 + 7 = A(4 - 4 + 5)$

$1 = 5A \Rightarrow A = \frac{1}{5}$

Coefficient of $s^2$

$2 = A + B$

$B = 2 - \frac{1}{5}$

$B = \frac{9}{5}$

Coeffi of $s$

$-7 = -2A + 2B + C$

$C = -7 + 2A + 2B$

$= -7 + \frac{2}{5} + \frac{18}{5}$

$C = -3$

$$y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{\frac{9}{5}s - 3}{s^2 - 2s + 5}$$

$$= \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s}{s^2-2s+5} - \frac{3}{s^2-2s+5}$$

$$= \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$= \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2+4} - \frac{1}{(s+1)^2+4} - \frac{3}{2}\frac{2}{(s+1)^2+2^2}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 4t - \frac{1}{2}e^{-t}\sin 2t - \frac{3}{2}e^{-t}\sin 2t$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}e^{-t}\cos 4t - 2e^{-t}\sin 2t$$

V) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ $\quad t=0,\ y=0,\ y'=2$

$y'' - 6y' + 8y = e^{3t}$

$s^2Y(s) - sy(0) - y'(0) - 6[sY(s) - y(0)] + 8Y(s) = \frac{1}{s-3}$

$s^2Y(s) - 0 - 2 - 6sY(s) + 0 + 8Y(s) = \frac{1}{s-3}$

$$y(s)[s^2 - 6s + 8] = \frac{1}{s-3} + 2$$

$$y(s) = \frac{1 + 2(s-3)}{s-3} \cdot \frac{1}{s^2-6s+8}$$

$$y(s) = \frac{1 + 2s - 6}{(s-3)(s^2-6s+8)}$$

$$y(s) = \frac{2s - 5}{(s-3)(s^2-6s+8)}$$

$$y(s) = \frac{2s-5}{(s-3)(s-4)(s-2)} \equiv \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$A: \left.\frac{2s-5}{(s-4)(s-2)}\right|_{s=3} = \frac{2(3)-5}{(-1)(1)}$$

$$= -1$$

$$B: \left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)}$$

$$= \frac{3}{2}$$

$$C: \left.\frac{2s-5}{(s-3)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(-1)(-2)}$$

$$= \frac{-1}{2}$$

$$y(s) = \frac{-1}{s-3} + \frac{3}{2} \cdot \frac{1}{s-4} - \frac{1}{2} \cdot \frac{1}{s-2}$$

$$= -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}$$