SUNNY OGHENERUME COLLINS
16/ENG04/070 ENG 381
ELECT/ELECT
ASSIGNMENT 5

(1) $\frac{dy}{dx} + 3y = e^{-2t}$ given that at $t=0$, $y=2$

$y' + 3y = e^{-2t}$

$y' = sy(s) - y(0)$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) = \frac{1}{s+2} + 2$

$sy(s) + 3y(s) = \frac{2s+5}{s+2}$

$y(s)(s+3) = \frac{2s+5}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$\frac{A}{s+2}\Big|_{s=-2} = \frac{2(-2)+5}{-2+3} = \frac{-4+5}{1} = \frac{+1}{1} = +1$

$\frac{B}{s+3}\Big|_{s=-3} = \frac{2(-3)+5}{-3+2} = \frac{-6+5}{-1} = 1$

$\therefore y(t) = L^{-1}\left[\frac{A}{s+2} + \frac{B}{s+3}\right]$

$L^{-1}\left[\frac{1}{s+2}\right] + L^{-1}\left[\frac{1}{s+3}\right]$

$y(t) = e^{2t} + e^{3t}$

(ii) $3\frac{dy}{dt} - 6y = \sin 2t$

at $t=0;\ y=1$

$3y' - 6y = \sin 2t$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$s(sy(s) - 1) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+2^2}$

$3sy(s) - 6y(s) = \frac{2}{s^2+4} + 3$

$3sy(s) - 6y(s) = \frac{3s^2+14}{s^2+4}$

$y(s)(3s-6) = \frac{3s^2+14}{s^2+4}$

$y(s) = \frac{3s^2+14}{(3s-6)(s+2)^2}$

(iii) $\frac{dy}{dt} - 4y = 8$ at $t=0;\ y=2$

$y' - 4y = 8$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = 8/s$

$sy(s) - 4y(s) = \frac{8}{s} + 2$

$sy(s) - 4y(s) = \frac{8 + 2s}{s}$

$y(s)(s-4) = \frac{2s+8}{s}$

$y(s) = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{A}{s}\Big|_{s=0} \quad \frac{2(0)+8}{0-4} = \frac{8}{4} = -2$

$\frac{B}{s-4}\Big|_{s=4} \quad \frac{2(4)+8}{4} = \frac{16}{4} = 4$

$y(t) = L^{-1}[y(s)]$

$= L^{-1}[-2/s + 4/s-4]$

$= L^{-1}[-2/s]\ L^{-1}[4/s-4]$

$= -2 + 4e^{4t}$

$= 4e^{4t} - 2$

(iv) $\frac{d^2y}{dt^2} - \frac{2dy}{dt} + 5y = e^{2t}$ at $t=0; y=2; y'=1$

$y'' - 2y' + 5y = e^{2t}$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) - 2 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) - 2s = \frac{1}{s-2} + 3$

$s^2y(s) - 2sy(s) + 5y(s) - 2s = \frac{3s-5}{s-2}$

$y(s)(s^2 - 2s - 2s + 5) = \frac{3s-5}{s-2}$

$y(s)(s^2 - 4s + 5) = \frac{3s-5}{s-2}$

$y(s) = \frac{3s-5}{(s^2-4s+5)(s-2)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-4s+5}$

$s^2 - 4s + 5$

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2}{2}$

$\Rightarrow 2 \pm 3$