Mat. NO: 15/ENG04/013

Math Assignment V (ENG.381)

① $\frac{dy}{dt} + 3y = e^{-2t}$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

At $t = 0$, $y = 2$

$$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{2(s+2)+1}{s+2}$$

$$y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$y(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$= \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$$

$$2s+5 = A(s+3) + B(s+2)$$

As $s \to -2$

$$-4+5 = A$$

$$A = 1$$

As $s \to -3$

$$-6+5 = -B$$

$$B = 1$$

$$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y(t) = e^{-2t} + e^{-3t}$$

② $3\frac{dy}{dt} - 6y = \sin 2t$

$$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$$

At $t = 0$, $y = 1$

$$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$$

$$y(s)[3s-6) = \frac{3(s^2+4)+2}{s^2+4}$$

$$y(s)(3s-6) = \frac{3s^2+14}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$y(s) = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$$

$$y(s) = \frac{(As+B)(3s-6) + C(s^2+4)}{(s^2+4)(3s-6)}$$

$$3s^2+14 = (As+B)(3s-6) + C(s^2+4)$$

As $s \to 2$

$$26 = 8C$$

$$C = \frac{26}{8} = \frac{13}{4}$$

Comparing Coefficients

$$3B = 0$$
$$B = 0$$

$$3As = 0$$
$$As = 0$$
$$A = 0$$

$$\therefore y(s) = \frac{13}{4}\left(\frac{1}{3s-6}\right)$$

$$= \frac{13}{4}\left[\frac{1}{3}\left(\frac{1}{s-2}\right)\right]$$

$$y(t) = \frac{13}{12}e^{2t}$$

(iii) $\frac{dy}{dt} - 4y = 8$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

At $t=0$, $y=2$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y(s)(s-4) = \frac{2s+8}{s}$

$y(s) = \frac{2s+8}{s(s-4)}$

$y(s) = \frac{A}{s} + \frac{B}{s-4}$

$= \frac{A(s-4) + Bs}{s(s-4)}$

$2s + 8 = A(s-4) + Bs$

As $s \to 0$

$8 = -4A$

$A = -2$

As $s \to 4$

$16 = 4B$

$B = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

$y(t) = -2(1 - 2e^{4t})$

(iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$s^2y(s) - sy(0) - y'(0) - 2[sy(s) - y(0)] - 5y(s) = \frac{1}{s-2}$

At $t = 0$, $y = 2$, $y' = 1$

$s^2y(s) - sy(0) - 1 - 2sy(s) + 4 - 5y(s) = 1/s-2$

$s^2y(s) - 2s - 2sy(s) + 3 - 5y(s) = 1/s-2$

$y(s)[s^2 - 2s - 5] = \frac{1}{s-2} + 2s - 3$

$y(s) = \frac{1 + 2s(s-2) - 3(s-2)}{(s-2)(s^2 - 2s - 5)}$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s+1.4)(s-3.4)}$$

$$y(s) = \frac{A}{s-2} + \frac{B}{s+1.4} + \frac{C}{s-3.4}$$

$$= \frac{A(s+1.4)(s-3.4) + B(s-2)(s-3.4) + C(s-2)(s+1.4)}{(s-2)(s+1.4)(s-3.4)}$$

$$2s^2 - 7s + 7 = A(s+1.4)(s-3.4) + B(s-2)(s-3.4) + C(s-2)(s+1.4)$$

As $s \to 2$

$$1 = -4.76A$$

$$A = \frac{-1}{4.76} = -0.2 \approx 0$$

As $s \to -1.4$

$$20.72 = 16.32A$$

$$A = \frac{20.72}{16.32} = 1.27 \approx 1$$

As $s \to 3.4$

$$6.32 = 6.72C$$

$$C = 0.94 \approx 1$$

Using Approximate values of all decimals

$$y(s) = \frac{1}{s+1} + \frac{1}{s-3}$$

$$y(t) = e^{-t} + e^{3t}$$

(v) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$$s^2y(s) - sy(0) - y'(0) - 6sy(s) - 6y(0) + 8y(s) = \frac{1}{s-3}$$

At $t = 0$, $y = 0$, $y' = 2$

$$s^2y(s) - 2 - 6sy(s) + 8y(s) = 1/s-3$$

$$y(s)[s^2 - 6s + 8] = \frac{2s-5}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = \frac{2s-5}{(s-3)(s+2)(s+4)}$$

$$y(s) = \frac{A}{s-3} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$= \frac{A(s+2)(s+4) + B(s-3)(s+4) + C(s-3)(s+2)}{(s-3)(s+2)(s+4)}$$

$$2s - 5 = A(s+2)(s+4) + B(s-3)(s+4) + C(s-3)(s+2)$$

As $s \to 3$

$$1 = 35A$$

$$A = 1/35$$

As $s \to -2$

$$-9 = -10B$$

$$B = 9/10$$

As $s \to -4$

$$-13 = 14C$$

$$C = -13/14$$

$$y(s) = \frac{1}{35}\left(\frac{1}{s-3}\right) + \frac{9}{10}\left(\frac{1}{s+2}\right) - \frac{13}{14}\left(\frac{1}{s+4}\right)$$

$$y(t) = \frac{1}{35}e^{3t} + \frac{9}{10}e^{-2t} - \frac{13}{14}e^{-4t}$$