USMAN AMEER NAT SAMRATH
IS(ENG03/0314
CIVIL ENGINEERING
ENG381.

1) $\dot{y}+3y = e^{-2t}$

$t=0, y=2$

$sy(s) + y(s) + 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) + 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$L^{-1}\left\{\frac{2s+5}{(s+2)(s+3)}\right\} = y(s)$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3)+B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3)+B(s+2)$

when $s=-2$.

$-4+5 = A(1)+0$

$A = 1$

when $s=-3$

$-6+5 = A(0)+B(-1)$

$\frac{+B}{+} = \frac{+1}{+}$

$B = 1$

$y(s) = L^{-1}\left[\frac{1}{s+2} - \frac{1}{s+3}\right]$

$y(s) e^{-2t} + 3^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0, y=1$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3y(s)(s-2) - 3 = \frac{2}{s^2+4}$

$3y(s)(s-2) = \frac{2}{s^2+4} + 3$

$\frac{2+3s^2+12}{s^2+4}$

$y(s) = \frac{3s^2+14}{3(s-2)(s^2+4)}$

$= \frac{3s^2+14}{(3s-6)(s^2+4)}$

$y(s) = L^{-1}\left[\frac{3s^2+14}{(3s-6)(s^2+4)}\right]$

$\frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}$

$3s^2+14 = A(s^2+4)+Bs+C(3s-6)$

at $s=2$

$12+14 = A(8)+(Bs+c)(0)$

$\frac{26}{8} = \frac{8A}{8}$

$A = 13/4$

Using the method of coefficient

$3 = A+B$ ---(1)

$3 = 13/4 + 3B$

$3B = 3 \cdot \frac{13}{4}$

$12 = 13$

$3B = -1/4 \quad B = -1/12$

$14 = 4A - 6c$

$14 = 4 \times \frac{13}{4} - 6c$

$14 = 13 - 6c$

$1 = -6c$

$c = -1/6$

$y(s) = L^{-1}\left[\frac{13}{14} \times \frac{1}{s-6} + \left(\frac{-1/12 s - 1/6}{s^2+4}\right)\right]$

$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-6} - \frac{1}{12} \times \frac{s}{s^2+4} - \frac{1}{6} \times \frac{1}{s^2+4}\right]$

$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-6}\right] + L^{-1}\left[-\frac{1}{12} \times \frac{s}{s^2+4}\right] + L^{-1}\left(\frac{1}{6} \times \frac{2}{s^2+4}\right)$

$= \frac{13}{12} e^{6t} - \frac{1}{12}\cos 2t + \frac{1}{12}\sin 2t$

3) $\dot{y} - 4y = 8$

at $t = 0$, $y = 2$.

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y(s)(s-4) = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$y(s) = L^{-1}\frac{8+2s}{s(s-4)} = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$\frac{8+2s}{s(s-4)} = \frac{A(s-4)}{s(s-4)} + \frac{B(s)}{s(s-4)}$

$8 + 2s = A(s-4) + B(s)$

when $s = 4$

$8 + 8 = A(0) + 4B$

$B = 4$

when $s = 0$

$8 = -4A + 0$

$\frac{-4A}{4} + \frac{8}{4}$

$A = -2$

$y(s) = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$

$y(s) = -2 + 4e^{4t}$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

at $t = 0$, $y = 2$, $y' = 1$

$s^2y(s) - s(2) - 1 - 2(sy(s) - 2) + 5y(s)$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s)$

$s^2y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2-2s+5) = \frac{1}{s-2} + 2s+3$

$\frac{1+2s^2-4+3s-6}{s-2}$

$y(s)\,s^2-2s+5 = \frac{2s^2+3s-9}{s-2}$

$y(s) = \frac{2s^2-13s-9}{(s-2)(s^2-2s+5)}$

$L^{-1}\frac{2s^2+3s-9}{(s-2)(s^2-2s+5)}$

$= \left[\frac{A}{(s-2)} + \frac{Bs+C}{s^2-2s+5}\right]$

$2s^2+3s-9 = A(s^2-2s+5) + (Bs+c)(s-2)$

at $s=2$

$8+6-9 = A(4-8+5)+0$

$A = 5$

Using the method of coefficient:

$2 = A+B$

$2 = 5+B$

$B = -3$

$3 = -2A-2B+C$

$3 = -2(5)-2(-3)+C$

$3 = -10+6+C$

$3 = -4+C$

$C = -7$

$y(s) = \left[\frac{5}{s-2} + \frac{-3s-7}{s^2-2s+5}\right]$

$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[\frac{3s}{(s-1)^2+4}\right]$

$- L^{-1}\left[\frac{7}{(s-1)^2+4}\right]$

$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[\frac{3\times s}{(s-1)^2+4}\right]$

$- L^{-1}\left[\frac{3}{2}\times\frac{2}{(s-1)^2+4}\right]$

$= 5e^{2t} - 3e^{t}\cos 2t - \frac{7}{2}e^{t}\sin 2t$

5) $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$

at $t=0$, $y=0$, $y' = 2$.

$\Rightarrow s^2y(s) - sy(0) - y'(0) - 6$
$(sy(s) - y(s)) + 8y(s)$

$\Rightarrow s^2y(s) - 0 - 2 - 6sy(s)$
$+ 6(0) + 8y(s) = 1/s-3$

$\Rightarrow s^2y(s) - 2 - 6sy(s) + 8y(s)$
$= 1/s-3$

$\Rightarrow y(s)(s^2-6s+8) = \frac{1}{s-3} + 2$

$y(s)(s^2-6s+8) = \frac{1+2s-6}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$

$y(s) = L^{-1}\left[\frac{2s-5}{(s-3)(s-4)(s-2)}\right]$

$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$

$\frac{2s-5}{(s-3)(s-4)(s-2)} \Rightarrow A(s-4)(s-2)$
$+B(s-3)(s-2)$
$+C(s-3)(s-4)$

$\frac{2s-5}{(s-3)(s-4)(s-2)} \quad (s-3)(s-2)(s-4)$

$2s-5 = A(s-4)(s-4) + B(s-3)(s-2)$
$+ C(s-3)(s-4)$

at $s=2$

$4-5 = A(0) + B(0) + C(-1)(-2)$

$-1 = 2C$

$C = -1/2$

at $s=3$

$6-5 = A(-1)(1) + B(0) + C(0)$

$1 = -A \quad \therefore A = -1$

at $s=4$: $8-5 = A(0)+B(2)(1)+C(0)$

$3 = 2B \quad \therefore B = 3/2$

$y(s) = L^{-1}\left[\frac{-1}{s-3} + \frac{3}{2}\times\frac{1}{s-4} - \frac{1}{2}\times\frac{1}{s-2}\right]$