$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$$

a) Rewriting eqn in terms of its transforms

$\mathcal{L}\{y\} = \bar{y}$

$\mathcal{L}\{\ddot{y}\} = s^2\bar{y} - sy_0 - y_1$

$\mathcal{L}\{\dot{y}\} = s\bar{y} - y_0$

$\mathcal{L}\{e^{3t}\} = \frac{1}{s-3}$

$s^2\bar{y} - sy_0 - y_1 - 6(s\bar{y} - y_0) + 8\bar{y} = \frac{1}{s-3}$

$s^2\bar{y} - sy_0 - y_1 - 6s\bar{y} + 6y_0 + 8\bar{y} = \frac{1}{s-3}$

b) Putting initial condition $t=0,\ y=0 = y_0,\ y' = 2 = y_1$

$s^2\bar{y} - 0 - 2 - 6s\bar{y} + 0 + 8\bar{y} = \frac{1}{s-3}$

$s^2\bar{y} - 6s\bar{y} + 8\bar{y} - 2 = \frac{1}{s-2}$

$(s^2 - 6s + 8) = \frac{1}{s-2} + \frac{2}{1}$

$(s-4)(s-2) = \frac{1 + 2(s-2)}{s-2}$

$\bar{y} = \frac{1 + 2s - 4}{(s-2)} = \frac{2s-3}{(s-2)} \times \frac{1}{(s-4)(s-2)}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ Emuchalu chris 15/ENG01/2006 ENG381

a) Resolving the equation above in laplace transforms, using dot notation

$$\mathcal{L}\{y\} = \bar{y}$$

$$\mathcal{L}\{\dot{y}\} = s\bar{y} - y_0$$

$$\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$$

The equation becomes

$$3(s\bar{y} - y_0) - 6\bar{y} = \frac{2}{s^2+4}$$

$$3s\bar{y} - 3y_0 - 6\bar{y} = \frac{2}{s^2+4}$$

b) Applying initial conditions i.e $t=0$, $y=1=y_0$

$$3s\bar{y} - 3(1) - 6\bar{y} = \frac{2}{s^2+4}$$

$$3s\bar{y} - 3 - 6\bar{y} = \frac{2}{s^2+4}$$

c) Rearranging to obtain $\bar{y}$

$$3s\bar{y} - 6\bar{y} = \frac{2}{s^2+4} + 3$$

$$\bar{y}(3s-6) = \frac{2}{s^2+4} + \frac{3}{1}$$

$$\bar{y}(3s-6) = \frac{2 + 3(s^2+4)}{s^2+4}$$

$$\bar{y}(3s-6) = \frac{2 + 3s^2 + 12}{(s^2+4)}$$

$$\bar{y}(3s-6) = \frac{3s^2+14}{(s^2+4)}$$

$$\bar{y} = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$\bar{y} = \frac{A}{(3s-6)} + \frac{Bs+c}{(s^2+4)}$$

$$3s^2+14 = A(s^2+4) + Bs+C(3s-6$$

$$3 = A+3B$$
$$14 = 4A-6c$$

when $c=2$

$$14+12 = A(4+4) + (Bs+c)(0)$$
$$26 = 8A$$
$$A = \frac{13}{8}$$
$$3 = \frac{13}{8} + 3B$$
$$3 - \frac{13}{8} = 3B$$
$$B = -\frac{1}{12}$$
$$14 = 4 \times \frac{13}{4} - 6c$$
$$14 = 13-6c$$
$$6c = -1$$
$$c = \frac{-1}{6}$$

$$y = \frac{\frac{13}{8}}{(3s-6)} + \frac{-\frac{1}{12}s - \frac{1}{6}}{(s^2+4)}$$

$$\bar{y} = \left[\frac{13}{8} \times \frac{1}{s-2} - \frac{1}{12} s \times \frac{1}{s^2+4} - \frac{1}{12} \frac{2}{s^2+4}\right]$$

$\therefore$ To obtain y

$$y = \mathcal{L}^{-1}\left[\frac{13}{8} \times \frac{1}{s-2} - \frac{1}{12} s \times \frac{1}{s^2+4} - \frac{1}{12} \frac{2}{s^2+4}\right]$$

$$y = \frac{13}{8} e^{2t} - \frac{1}{12} \cos 2t - \frac{1}{12} \sin 2t.$$

3) $\frac{dy}{dt} - 4y = 8$ $\quad t=0, y=2$

a) Resolving the equation above in laplace transforms, using dot notation.

$\mathcal{L}\{y\} = \bar{y}$

$\mathcal{L}\{\dot{y}\} = s\bar{y} - y_0$

$\mathcal{L}\{8\} = \frac{8}{s}$

The equation becomes

$$s\bar{y} - y_0 - 4\bar{y} = \frac{8}{s}$$

b) Applying initial conditions $t=0, y=2=y_0$

$$s\bar{y} - 2 - 4\bar{y} = \frac{8}{s}$$

$$s\bar{y} - 4\bar{y} - 2 = \frac{8}{s}$$

c) Rearranging to obtain value of $\bar{y}$

$$\bar{y}(s-4) = \frac{8}{s} + 2$$

$$\bar{y}(s-4) = \frac{8+2s}{s}$$

$$\bar{y} = \frac{8+2s}{s(s-4)}$$

$$\bar{y} = \frac{2s+8}{s(s-4)}$$

d) Taking inverse transform to obtain y

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$\frac{2s+8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$$

$$2s+8 = A(s-4) + Bs$$

$$2s+8 = As + Bs - 4A$$

$$2s+8 = s(A+B) - 4A$$

$A+B=2$

$-4A = 8$

$A = -2$

$-2 + B = 2$

$B = 4$

$$\bar{y} = \frac{-2}{s} + \frac{4}{s-4}$$

$$\bar{y} = \frac{4}{s-4} - \frac{2}{s}$$

Taking inver $\mathcal{L}^{-1}\left[\frac{4}{s-4} - \frac{2}{s}\right]$

$$y = 4e^{4t} - 2 \quad //$$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

a) Finding the Laplace transforms using <u>dot notation</u>

$\mathcal{L}\{\ddot{y}\} = s^2\bar{y} - sy_0 - y_1$

$\mathcal{L}\{\dot{y}\} = s\bar{y} - y_0$

$\mathcal{L}\{y\} = \bar{y}$

$\mathcal{L}\{e^{2t}\} = \frac{1}{s+2}$ equation becomes $s^2\bar{y} - sy_0 - y_1 - 2(s\bar{y} - y_0) + 5\bar{y} = \frac{1}{s-2}$

b) Rearranging to obtain $\bar{y}$

$$s^2\bar{y} - sy_0 - y_1 - 2s\bar{y} + 2y_0 + 5\bar{y} = \frac{1}{s-2}$$

Putting initial conditions

$y = y_0 = 2$, $y' = 1 = y_1$

$$s^2\bar{y} - 2s - 1 - 2s\bar{y} + 4 + 5\bar{y} = \frac{1}{s-2}$$

$$s^2\bar{y} - 2s\bar{y} + 5\bar{y} + 3 = \frac{1}{s-2}$$

$$\bar{y}(s^2 - 2s + 5) = \frac{1}{s-2} - 3$$

$\cancel{1-3(s-2)}$ $\frac{7-3s}{s-3} = \bar{y}(s^2 - 2s + 5)$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

a) Finding the Laplace transforms using dot notation

$\mathcal{L}\{\ddot{y}\} = s^2\bar{y} - sy_0 - y_1$

$\mathcal{L}\{\dot{y}\} = s\bar{y} - y_0$

$\mathcal{L}\{y\} = \bar{y}$

$\mathcal{L}\{e^{2t}\} = \frac{1}{s+2}$, equation becomes $s^2\bar{y} - sy_0 - y_1 - 2(s\bar{y} - y_0) + 5\bar{y}$

b) Rearranging to obtain $\bar{y}$

$s^2\bar{y} - sy_0 - y_1 - 2s\bar{y} + 2y_0 + 5\bar{y} = \frac{1}{s-2}$

Putting initial conditions

$y = y_0 = 2$, $y' = 1 = y_1$

$s^2\bar{y} - 2s - 1 - 2s\bar{y} + 4 + 5\bar{y} = \frac{1}{s-2}$

$s^2\bar{y} - 2s\bar{y} + 5\bar{y} + 3 = \frac{1}{s-2}$

$\bar{y}(s^2 - 2s + 5) = \frac{1}{s-2} - 3$

$\cancel{1-3(s-2)}$ $\frac{7-3s}{s-3} = \bar{y}(s^2 - 2s + 5)$

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i) $\frac{dy}{dt} + 3y = e^{-2t}$

a) Resolving the equation above in Laplace transforms, using dot notation

$\mathcal{L}\{y\} = \bar{y}$

$\mathcal{L}\{\dot{y}\} = s\bar{y} - y_0$

$\mathcal{L}\{e^{-2t}\} = \frac{1}{s+2}$

The equation becomes

$s\bar{y} - y_0 + 3\bar{y} = \frac{1}{s+2}$

b) Applying initial conditions ie $t=0,\ y=2=y_0$

$s\bar{y} - 2 + 3\bar{y} = \frac{1}{s+2}$

c) Rearranging to obtain $\bar{y}$

$s\bar{y} + 3\bar{y} - 2 = \frac{1}{s+2}$

$s\bar{y} + 3\bar{y} = 2 + \frac{1}{s+2}$

$\bar{y}(s+3) = \frac{2(s+2)+1}{s+2}$

$\bar{y}(s+3) = \frac{2s+4+1}{(s+2)}$

$\bar{y}(s+3) = \frac{2s+5}{(s+2)}$

$\bar{y} = \frac{2s+5}{(s+2)(s+3)}$

d) Find Inverse to obtain y

Find Partial fraction first

$\bar{y} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$= \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$,

$$\bar{y} = 2s+5 = A(s+3) + B(s+2)$$

$$2s+5 = As + 3A + Bs + 2B$$

$$2s+5 = As + Bs + 3A + 2B$$

$$2s+5 = s(A+B) + 3A + 2B$$

$$A+B = 2 \quad \times 2$$

$$3A+2B = 5 \quad \times 1$$

$$\cancel{2}A + \cancel{2}B = 4 \qquad A+B=2$$

$$3A + \cancel{2B} = 5 \qquad 1+B=2$$

$$-A = -1 \qquad B = 2-1$$

$$A = 1 \qquad B = 1$$

$$\bar{y} = \frac{1}{(s+2)} + \frac{1}{(s+3)}$$

Laplace inverse to obtain y

$$\mathcal{L}^{-1}\left[\frac{1}{(s+2)} + \frac{1}{(s+3)}\right]$$

$$y = e^{-2t} + e^{-3t}$$ //