(1)

$\frac{dy}{dt} + 3y = e^{-2t}$

$t=0, y=2,$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) = \frac{1}{s+2} + 2$

$sy(s) + 3y(s) = \frac{2+2s+4}{s+2} = \frac{2s+1}{s+2}$

$y(s)(s+3) = \frac{2s+1}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{A}{s+2} + \frac{B}{s+3} = \frac{2s+5}{(s+2)(s+3)}$

$A(s+3) + B(s+2) = 2s+5$

let $s = -2$

$A = 1$

let $s = -3$

$B = 1$

$\mathcal{L}\left[\frac{1}{s+2} + \frac{1}{s+3}\right] = e^{-2t} + e^{3t}$

(2)

(ii) $3\frac{dy}{dt} - 6y = \sin 2t$

$t=0, y=1$

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s+4}$

$$3sy(s) - 3(1) - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 6y(s) = \frac{2}{s^2+4} + 3$$

$$3sy(s) - 6y(s) = \frac{3s^2+14}{s^2+4}$$

$$y(s)(3s-6) = \frac{3s^2+14}{s^2+4)(3s-6)}$$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A}{3s-6} + \frac{Bs+C}{s^2+4}$$

$$3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$$

Let $s = 2$

$$26 = 8A$$

$$A = 13/4$$

Comparing co-efficients.

$$3 = A + 3B$$

$$3 = 13/4 + 3B$$

$$-1/4 = 3B$$

$$B = -1/12$$

$$14 = 4A - 6C$$

$$14 = 4 \cdot 13/4 - 6C$$

$$14 = 13 - 6C$$

$$-6C = 14 - 13$$

$$-6C = 1$$

$$C = -1/6$$

$$L\left\{\frac{13/4}{3s-6} + \frac{(-1/12)s - 1/6}{s^2+4}\right\}$$

$$L\left\{\frac{13/12 \cdot 1}{s-2} - \frac{1/12 \cdot s}{s^2+2^2} - \frac{1/12 \cdot 2}{s^2+2}\right\}$$

$$= 13/12\, e^{2t} - 1/12 \cos 2t - 1/12 \sin 2t$$

(11) $\frac{dy}{dt} - 4y = 8$ given that $t=0, y=2$

$sy(s) - s(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$sy(s) - 4y(s) = \frac{8}{s} + 2$

$sy(s) - 4y(s) = \frac{8+2s}{s}$

$y(s)[s-4] = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$\frac{A}{s} + \frac{B}{s-4} = \frac{8+2s}{s(s-4)}$

$A(s-4) + B(s) = 8+2s$

Let $s=0$

$-4A = 8$

$A = -2$

Les $s = +4$

$4B = 8+8$

$4B = 16$

$B = 4$

$L\left\{\frac{-2}{s} + \frac{4}{s-4}\right\} = -2 + 4e^{4t}$

(12)

$\frac{d^2y}{dt} - 2\frac{dy}{dt} + 5y = e^{2t}$

$t=0, y=2, y'=1$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s = 1 - 2\{sy(s) - 2\} + 5y$

$s^2y(s) - 2s - 1 - 2\{sy(s) - 2\} + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s + 3 - 2sy(s) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) = \frac{1}{s-2} + 2s - 3$

$y(s)\{s^2 - 2s + 5\} = \frac{1 + 2s^2 - 7s + 6}{s-2}$

$y(s) = \frac{2s^2 - 7s + 6}{(s-2)(s^2 - 2s + 5)}$

$\frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5} = \frac{2s^2 - 7s + 1}{(s-2)(s^2 - 2s + 5)}$

$A(s^2 - 2s + 5) + BS - B(s-2) = 2s^2 - 7s + 7$

let $s = 2$

$A(5) = 1$

$A = 1/5$

comparing co-efficients

$A + B = 2$

$1/5 + B = 2$

$B = 2 - 1/5$

$B = 9/5$

$5A + 2C = 7$

$5 \cdot 1/5 + 2C = 7$

$1 + 2C = 7$

$2C = 6$

$C = 3$

$L\left\{\frac{1/5}{s-2} + \frac{(-9/5) + 3}{s^2 - 2s + 5}\right\} = L\left[\frac{1/5}{s-2} + \frac{(-9/5)}{s^2 - 2s + 5} + \frac{3}{s^2 - 2s + 5}\right]$

$$L^{-1}\left\{\frac{1/5}{s-2} - \frac{9/5}{(s-2)^2+4} + \frac{3}{(s-2)+4}\right\}$$

$$L^{-1}\left\{\frac{1/5}{s-2} - \frac{9/5\,s}{\{s-2\}^2+2^2} + \frac{2}{(s-2)^2+2^2}\cdot\frac{3}{2}\right\}$$

$$= \frac{1}{5}e^{2t} - \frac{9}{5}e^{2t}\cos 2t + \frac{3}{2}e^{2t}\sin 2t$$

(5)

(1) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$t=0, y=0, y'=0$

$s^2y(s) - sy(0) - y'(0) - 6\{sy(s) - y(0)\} + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - s\{0\} - 2 = 6\{sy(s) - 0\}$

$s^2y(s) - s\{0\} - 2 - 6\{sy(s) - 0\} + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 2 - 6\{sy(s)\} + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) = \frac{1}{s-3} + 2$

$y(s)\{s^2 - 6s + s\} = \frac{2s-5}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$A(s-2) + B(s-4) = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-4) = 2s$$

let $s = 3$

$$A(-2) = -3$$

$$A = \frac{-3}{-2}$$

$$A(-1) = 1$$

$$A = -1$$

let $s = 4$

$$2B = 3$$

$$B = 3/2$$

let $s = 2$

$$2C = -1$$

$$C = -1/2$$

$$\mathcal{L}\left\{ \frac{-1}{s-3} + \frac{3/2}{s-4} - \frac{1/2}{s-2} \right\}$$

$$y(s) = -e^{3t} + 3/2\, e^{4t} - 1/2\, e^{2t}$$