ASSIGNMENT [5]

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15/ENG/C2/0174

COMPUTER ENGINEERING

ENG381

1. $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0$, $y=2$

$sY(s) - y(0) + 3Y(s) = 1/(s+2)$

$sY(s) - 2 + 3Y(s) = 1/(s+2)$

$Y(s)(s+3) = 2/1 + 1/(s+2)$

$Y(s)(s+3) = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+3)(s+2)} = \frac{A}{s+3} + \frac{B}{s+2}$

$A(s+3)|_{s=-3} = \frac{2s+5}{s+2} = \frac{2(-3)+(-3)}{(-3+2)} = 9$

$B(s+2)|_{s=-2} = \frac{2s+5}{s+3} = \frac{2(-2)+(-2)}{(-2+3)} = -6$

$= \frac{9}{s+3} - \frac{6}{s+2}$

$= 9e^{-3t} - 6e^{-2t}$

2. $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t=0$, $y=1$

$3(sY(s) - y(0)) - 6Y(s) = 2/(s^2+4)$

$3(sY(s) - 1) - 6Y(s) = 2/(s^2+4)$

$3(sY(s) - 6Y(s) = 1/1 + 2/(s^2+4)$

$3Y(s)(s-6) = \frac{(s+2)^2 + 2}{(s+2)^2}$

$Y(s) = \frac{(s^2+4)+2}{(s-6)(s+2)^2} = \frac{A}{s-6} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$Y(s) = \frac{(s^2+4)+2}{(s-6)(s+2)^2} = \frac{A(s+2)^2 + B(s-6)(s+2) + C(s-6)}{(s-6)(s+2)^2}$

$(s^2+4)+2 = A(s+2)^2 + B(s-6)(s+2) + C(s-6)$

$s = -2$ $\quad$ $s = 6$ $\quad$ $s = 0$

$10/-8 = -8/-8\,C$ $\quad$ $42/64 = 64/64\,A$ $\quad$ $6 = 4A - 12B - 6C$

$C = -5/4$ $\quad$ $A = 21/32$ $\quad$ $6 = 4(21/32) - 12B - 6(-5/4)$

$6 = 84/32 + 30/4 - 12B$

$6 = 81/8 - 12B$

$6 - 81/8 = -12B$

$-33/8 = B \quad \therefore B = 11/32$

$= 21/32\left[1/(s-6)\right] + 11/32\left(1/(s+2)\right) - 5/4\left(1/(s^2+4)\right)$

$= \frac{21}{32}e^{6t} + \frac{11}{32}e^{-2t} - \frac{5}{4}\sin 2t$

3. $\frac{dy}{dt} - 4y = 8$ given that at $t=0$, $y=2$

$s(y) - y(0) - 4y(s) = 8/s$

$s(y) - 2 - 4y(s) = 8/s$

$y(s)(s-4) = 8/s + 2/1 = \frac{2s+8}{s}$

$y(s) = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$s = 4$ $\quad$ $s = 3$

$-10(4) + 31 = B(4-2)(4-3)$ $\quad$ $-10(3) + 31 = C(2-2)(3-4)$

$B = \frac{-9}{2}$ $\quad$ $1 = -C$

$C = -1$

$s = 2$

$-10(2) + 31 = A(2-4)(2-3)$

$A = \frac{11}{2}$

$= \left(\frac{11}{2(s-2)} - \frac{9}{2(s-4)}\right) - \frac{1}{s-3}$

$= \frac{1}{2}\left(\frac{11}{s-2} - \frac{9}{s-4}\right) - \frac{1}{s-3}$

$= \frac{1}{2}(11e^{2t} - 9e^{4t}) - e^{3t} = \frac{1}{2}(11e^{2t} - 9e^{4t}) - e^{3t}$

4. $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ given that at $t=0$, $y=2$, $y'=1$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$

$y(s)\cdot(s^2 - 2s + 5) = 2s - 3 + \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 7}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s^2-2s+5)(s-2)} = \frac{2s^2 - 7s + 7}{(s-1)(s-3)(s-2)} = \frac{A}{s-1} + \frac{B}{s-3} + \frac{C}{s-2}$

$A = \left.\frac{2s^2-7s+7}{(s-3)(s-2)}\right|_{s=1} = \frac{2(1)^2 - 7(1) + 7}{(1-3)(1-2)} = \frac{2}{2} = 1$

$B = \left.\frac{2s^2-7s+7}{(s-1)(s-2)}\right|_{s=3} = \frac{2(3)^2 - 7(3) + 7}{(3-1)(3-2)} = \frac{4}{2} = 2$

$C = \left.\frac{2s^2-7s+7}{(s-1)(s-3)}\right|_{s=2} = \frac{2(2)^2 - 7(2) + 7}{(2-1)(2-3)} = \frac{1}{-1} = -1$

$= \frac{1}{s-1} + \frac{2}{s-3} - \frac{1}{s-2}$

$e^t + 2e^{3t} - e^t$ //

5. $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ given that at $t=0$, $y=0$, $y'=2$

$s^2y(s) - sy(0) - y(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 0 - 2 - 6sy(s) + 12 + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) + 10 = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) = \frac{10}{1} + \frac{1}{s-3} = \frac{-10s + 31}{s-3}$

$y(s) = \frac{-10s + 31}{(s^2-6s+8)(s-3)} = \frac{10s + 31}{(s-2)(s-4)(s-3)}$

$y(s) = \frac{-10s + 31}{(s^2-6s+8)(s-3)} = \frac{A}{s-2} + \frac{B}{s-4} + \frac{C}{s-3}$

$A(s-2)\left|_{s=2}\right. = \frac{10s + 31}{(s-4)(s-3)} = \frac{10(2) + 31}{(2-4)(2-3)} = \frac{51}{2}$

$B(s-4)\left|_{s=4}\right. = \frac{10s+31}{(s-2)(s-3)} = \frac{10(4) + 31}{(4-2)(4-3)} = \frac{71}{2}$

$$C(s-3)\Big|_{s=3} = \frac{10s+31}{(s-2)(s-4)} = \frac{10(3)+31}{(3-2)(3-4)} = \frac{61}{-1} = -61$$

$$= \frac{51/2}{s-2} + \frac{71/2}{s-4} - \frac{61}{s-3}$$

$$= \frac{51}{2}e^{2t} + \frac{71}{2}e^{4t} - 61e^{3t}$$ //