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i) $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t = 0$, $y = 2$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

at $t=0$ $y(0) = 2$

$$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$$

$$y(s)(s+3) - 2 = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{1}{s+2} + 2$$

$$y(s)(s+3) = \frac{1 + 2(s+2)}{s+2}$$

$$y(s) = \frac{1 + 2s + 4}{(s+2)(s+3)}$$

$$y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$y(t) = L^{-1}\left[\frac{2s+5}{(s+2)(s+3)}\right]$$

$$y(t) = L^{-1}\left[\frac{A}{s+2} + \frac{B}{s+3}\right]$$

$$2s + 5 = A(s+3) + B(s+2)$$

at $s = -3$

$$2(-3) + 5 = -B$$

$$-1 = -B$$

$$\therefore B = 1$$

at $s = -2$

$$2(-2) + 5 = A$$

$$1 = A$$

$$\therefore A = 1$$

$\therefore y(t) = L^{-1}\left[\frac{1}{S+2} + \frac{1}{S+3}\right]$

$= L^{-1}\left[\frac{1}{S+2} + \frac{1}{S+3}\right]$

$\therefore y(t) = e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t=0, y=1$

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

at $t=0$ $y(0) = 1$

$3sy(s) - 3(1) - 6y(s) = \frac{2}{s^2+4}$

$y(s)[3s - 6] - 3 = \frac{2}{s^2+4}$

$(3s-6)y(s) = \frac{2}{s^2+4} + 3$

$y(s)(3s-6) = \frac{2+3(s^2+4)}{s^2+4}$

$y(s) = \frac{2+3s^2+12}{(3s-6)(s^2+4)}$

$y(t) = L^{-1}\left[\frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}\right]$

$\therefore 2+3s^2+12 = A(s^2+4) + Bs+C(3s-6)$

~~2 - 3s^2 + 12~~ At $s = 2$

$2+3(2)^2+12 = A(2^2+4) + 0$

$2+12+12 = 8A$

$26 = 8A$

$A = 13/4$

$\therefore 2 + 3s^2 + 12 = As^2 + 4A + 3Bs^2 - 6Bs + 3Cs - 6C$

$\therefore A + 3B = +3$

$\frac{13}{4} + 3B = +3$

$$3B = +3 - \frac{13}{4}$$

$$3B = \frac{12-13}{4}$$

$$\frac{3B}{3} = \frac{-1}{4} \times \frac{1}{3}.$$

$$B = \frac{-1}{12},$$

$$\therefore 4A - 6C = 14$$

$$4\left(\frac{13}{4}\right) - 6C = 14$$

$$13 - 6C = 14$$

$$-6C = -13 + 14$$

$$-6C = 1$$

$$C = \frac{-1}{6}$$

$$\therefore y(t) = L^{-1}\left[\frac{13/4}{(3s-6)} + \frac{-1/12\, s + (-1/6)}{(s^2+4)}\right)$$

$$y(t) = L^{-1}\left[\frac{13}{4}\frac{1}{3(s-2)} - \frac{1}{12}\frac{s}{s^2+4} - \frac{1}{6}\frac{1}{s^2+4}\right]$$

$$y(t) = L^{-1}\left[\frac{13}{12}\frac{1}{(s-2)} - \frac{1}{12}\frac{s}{s^2+4} - \frac{1}{12}\frac{2}{s^2+4}\right]$$

$$y(t) = \frac{1}{12} L^{-1}\left[\frac{13}{(s-2)} - \frac{s}{(s^2+4)} - \frac{2}{(s^2+4)}\right)$$

$$y(t) = \frac{1}{12}\left[13e^{2t} - \cos 2t - \sin 2t\right]$$

3) $\frac{dy}{dt} - 4y = 8$ at $t=0$ $y=2$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

at $t=0$ $y(0)=2$.

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y(s)(s-4) - 2 = \frac{8}{s}$

$y(s)(s-4) = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$y(t) = L^{-1}\left[\frac{2s+8}{s(s+4)}\right] = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$\cancel{y(t)}$ $2s+8 = A(s-4) + Bs$

at $s=0$

$2(0)+8 = A(0-4)+0$

$8 = -4A$

$A = -2$

at $s=4$

$2(4)+8 = 4B$

$16 = 4B$

$\therefore B = 4$

$y(t) = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$

$y(t) = -2 + 4e^{4t}$

$y(t) = 4e^{4t} - 2$

15/ENG02/012.

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0$ $y=2$ $y'=1$

$\therefore S^2 y(s) - S y(0) - y'(0) - 2\left[S y(s) - y(0)\right] + 5y(s) = \frac{1}{S-2}$

at $t=0$ $y(0)=2$ $y'(0)=1$

$S^2 y(s) - S(2) - 1 - 2\left(S y(s) - 2\right] + 5y(s) = \frac{1}{S-2}$

$S^2 y(s) - 2S - 1 - 2S y(s) + 4 + 5y(s) = \frac{1}{S-2}$

$y(s)\left[S^2 - 2S + 5\right] - 2S - 1 + 4 = \frac{1}{S-2}$

$y(s)\left[S^2 - 2S + 5\right] = \frac{1}{S-2} + 2S - 3$

$y(s)(S^2 - 2S + 5) = \frac{1 + 2S(S-2) - 3(S-2)}{S-2}$

$y(s)\left(S^2 - 2S + 5\right) = \frac{1 + 2S^2 - 4S - 3S + 6}{S-2}$

$y(s) = \frac{2S^2 - 7S + 7}{(S-2)(S^2 - 2S + 5)}$

$\therefore y(t) = L^{-1}\left[\frac{A}{S-2} + \frac{BS + C}{S^2 - 2S + 5}\right]$

$2S^2 - 7S + 7 = A(S^2 - 2S + 5) + BS + C(S-2)$

~~$2S^2 - 7S$~~ at $S=2$

$2(2)^2 - 7(2) + 7 = A(2^2 - 2(2) + 5)$

$8 - 14 + 7 = 5A$

$1 = 5A$

$A = 1/5$

$\therefore 2S^2 - 7S + 7 = AS^2 - 2AS + 5A + BS^2 - 2BS + CS - 2C$

$\therefore 5A - 2C = 7$

$5(1/5) - 2C = 7$

$-2C = 7 + 1$

$-2C = 8$ $\qquad C = -4$ //

$$-2A - 2B + C = -7$$

$$-2\left(\tfrac{1}{5}\right) - 2B - 4 = -7$$

$$\frac{-2}{5} - 2B - 4 = -7$$

$$-2B = -7 + 4 + \frac{2}{5}$$

$$-2B = \frac{-35 + 20 + 2}{5}$$

$$-2B = \frac{-13}{5}$$

$$B = \frac{13}{10}$$

$$\therefore y(t) = L^{-1}\left[\frac{1}{5}\frac{1}{s-2} + \frac{\tfrac{13}{10}s + 4}{s^2 - 2s + 5}\right]$$

$$y(t) = L^{-1}\left[\frac{1}{5}\frac{1}{s-2} + \frac{13}{10}\frac{1}{s^2 - 2s + 5} - \frac{4}{s^2 - 2s + 5}\right]$$

$$\cancel{y(t) = \frac{1}{5}e^{2t} + \frac{13}{10}\sin 2t\, exp}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{13}{10}\frac{\sin 2t\, e^{t}}{2} - \frac{4e^{t}\sin 2t}{2}$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{13}{20}e^{t}\sin 2t - 2e^{t}\sin 2t$$

$$y(t) = 4e^{2t} + 13e^{t}\sin 2t - 40e^{t}\sin 2t$$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$t=0,\ y=0,\ y'=2$

$s^2 y(s) - s y(0) - y'(0) - 6(s y(s) - y(0)) + 8y(s) = \frac{1}{s-3}$

at $t=0$ $y(0)=0$ $y'(0)=2$

$s^2 y(s) - 0 - 2 - 6 s y(s) - \cancel{6(0)} + 8y(s) = \frac{1}{s-3}$

$s^2 y(s) - 6 s y(s) + 8y(s) = \frac{1}{s-3} + 2$

$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$

$y(s) = \frac{1 + 2s - 6}{(s-3)(s^2 - 6s + 8)}$

$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)}$

$y(t) = L^{-1}\left[\frac{2s-5}{(s-3)(s-2)(s-4)}\right]$

$y(t) = L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}\right]$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

at $s=2$

$4-5 = B(2-3)(2-4)$

$-1 = 2B$

$\therefore B = -1/2$

at $s=3$

$6-5 = A(3-2)(3-4)$

$1 = -A$

$A = -1$

at $s = 4$

$$8 - 5 = C(4-3)(4-2)$$

$$3 = 2C$$

$$C = 3/2.$$

$$\therefore y(t) = L^{-1}\left[\frac{-1}{(s-3)} + \frac{-1/2}{(s-2)} + \frac{3/2}{(s-4)}\right]$$

$$y(t) = L^{-1}\left[\frac{-1}{(s-3)} - \frac{1}{2}\frac{1}{(s-2)} + \frac{3}{2}\frac{1}{(s-4)}\right]$$

$$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$

$$y(t) = -2e^{3t} - e^{2t} + 3e^{4t}$$

$$y(t) = 3e^{4t} - 2e^{3t} - e^{2t}$$